CHAPTER 1
Functions

EXERCISE SET 1.1

1. **(a)** around 1943 **(b)** 1960; 4200
(c) no; you need the year's population **(d)** war; marketing techniques
(e) news of health risk; social pressure, antismoking campaigns, increased taxation

2. **(a)** 1989; $35,600 **(b)** 1975, 1983; $32,000
(c) the first two years; the curve is steeper (downhill)

3. **(a)** $-2.9, -2.0, 2.35, 2.9$ **(b)** none **(c)** $y = 0$
(d) $-1.75 \le x \le 2.15$ **(e)** $y_{\max} = 2.8$ at $x = -2.6$; $y_{\min} = -2.2$ at $x = 1.2$

4. **(a)** $x = -1, 4$ **(b)** none **(c)** $y = -1$
(d) $x = 0, 3, 5$ **(e)** $y_{\max} = 9$ at $x = 6$; $y_{\min} = -2$ at $x = 0$

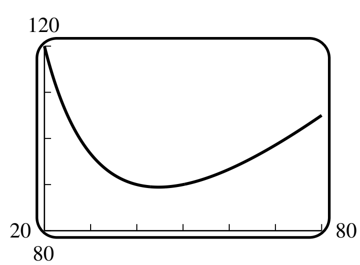
5. **(a)** $x = 2, 4$ **(b)** none **(c)** $x \le 2$; $4 \le x$ **(d)** $y_{\min} = -1$; no maximum value

6. **(a)** $x = 9$ **(b)** none **(c)** $x \ge 25$ **(d)** $y_{\min} = 1$; no maximum value

7. **(a)** Breaks could be caused by war, pestilence, flood, earthquakes, for example.
(b) C decreases for eight hours, takes a jump upwards, and then repeats.

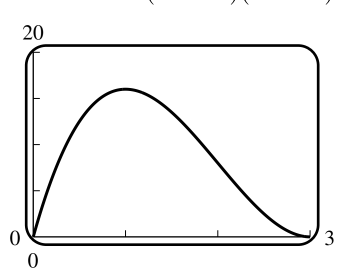
8. **(a)** Yes, if the thermometer is not near a window or door or other source of sudden temperature change.
(b) No; the number is always an integer, so the changes are in movements (jumps) of at least one unit.

9. **(a)** The side adjacent to the building has length x, so $L = x + 2y$. Since $A = xy = 1000$, $L = x + 2000/x$.
(b) $x > 0$ and x must be smaller than the width of the building, which was not given.
(c)



(d) $L_{\min} \approx 89.44$ ft

10. **(a)** $V = lwh = (6 - 2x)(6 - 2x)x$ **(b)** From the figure it is clear that $0 < x < 3$.
(c)

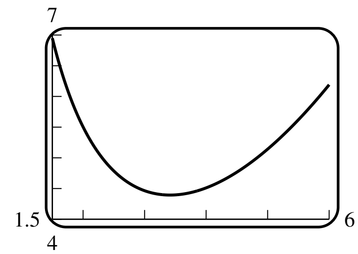


(d) $V_{\max} \approx 16$ in^3

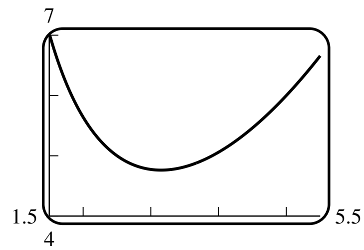
11. **(a)** $V = 500 = \pi r^2 h$ so $h = \dfrac{500}{\pi r^2}$. Then

$C = (0.02)(2)\pi r^2 + (0.01)2\pi rh = 0.04\pi r^2 + 0.02\pi r\dfrac{500}{\pi r^2}$

$= 0.04\pi r^2 + \dfrac{10}{r}$; $C_{\min} \approx 4.39$ at $r \approx 3.4$, $h \approx 13.8$.



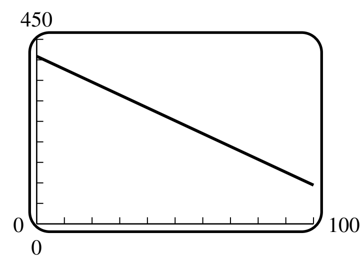
(b) $C = (0.02)(2)(2r)^2 + (0.01)2\pi rh = 0.16r^2 + \dfrac{10}{r}$. Since $0.04\pi < 0.16$, the top and bottom now get more weight. Since they cost more, we diminish their sizes in the solution, and the cans become taller.



(c) $r \approx 3.1$ cm, $h \approx 16.0$ cm, $C \approx 4.76$ cents

12. **(a)** The length of a track with straightaways of length L and semicircles of radius r is $P = (2)L + (2)(\pi r)$ ft. Let $L = 360$ and $r = 80$ to get $P = 720 + 160\pi = 1222.65$ ft. Since this is less than 1320 ft (a quarter-mile), a solution is possible.

(b) $P = 2L + 2\pi r = 1320$ and $2r = 2x + 160$, so
$L = \frac{1}{2}(1320 - 2\pi r) = \frac{1}{2}(1320 - 2\pi(80 + x))$
$= 660 - 80\pi - \pi x$.



(c) The shortest straightaway is $L = 360$, so $x = 15.49$ ft.

(d) The longest straightaway occurs when $x = 0$, so $L = 660 - 80\pi = 408.67$ ft.

EXERCISE SET 1.2

1. **(a)** $f(0) = 3(0)^2 - 2 = -2$; $f(2) = 3(2)^2 - 2 = 10$; $f(-2) = 3(-2)^2 - 2 = 10$; $f(3) = 3(3)^2 - 2 = 25$; $f(\sqrt{2}) = 3(\sqrt{2})^2 - 2 = 4$; $f(3t) = 3(3t)^2 - 2 = 27t^2 - 2$

(b) $f(0) = 2(0) = 0$; $f(2) = 2(2) = 4$; $f(-2) = 2(-2) = -4$; $f(3) = 2(3) = 6$; $f(\sqrt{2}) = 2\sqrt{2}$; $f(3t) = 1/3t$ for $t > 1$ and $f(3t) = 6t$ for $t \le 1$.

2. **(a)** $g(3) = \dfrac{3+1}{3-1} = 2$; $g(-1) = \dfrac{-1+1}{-1-1} = 0$; $g(\pi) = \dfrac{\pi+1}{\pi-1}$; $g(-1.1) = \dfrac{-1.1+1}{-1.1-1} = \dfrac{-0.1}{-2.1} = \dfrac{1}{21}$; $g(t^2-1) = \dfrac{t^2-1+1}{t^2-1-1} = \dfrac{t^2}{t^2-2}$

(b) $g(3) = \sqrt{3+1} = 2$; $g(-1) = 3$; $g(\pi) = \sqrt{\pi+1}$; $g(-1.1) = 3$; $g(t^2-1) = 3$ if $t^2 < 2$ and $g(t^2-1) = \sqrt{t^2-1+1} = |t|$ if $t^2 \ge 2$.

3. **(a)** $x \ne 3$ **(b)** $x \le -\sqrt{3}$ or $x \ge \sqrt{3}$

(c) $x^2 - 2x + 5 = 0$ has no real solutions so $x^2 - 2x + 5$ is always positive or always negative. If $x = 0$, then $x^2 - 2x + 5 = 5 > 0$; domain: $(-\infty, +\infty)$.

(d) $x \ne 0$ **(e)** $\sin x \ne 1$, so $x \ne (2n + \frac{1}{2})\pi$, $n = 0, \pm 1, \pm 2, \ldots$

4. **(a)** $x \neq -\frac{7}{5}$

(b) $x - 3x^2$ must be nonnegative; $y = x - 3x^2$ is a parabola that crosses the x-axis at $x = 0, \frac{1}{3}$ and opens downward, thus $0 \leq x \leq \frac{1}{3}$

(c) $\dfrac{x^2 - 4}{x - 4} > 0$, so $x^2 - 4 > 0$ and $x - 4 > 0$, thus $x > 4$; or $x^2 - 4 < 0$ and $x - 4 < 0$, thus $-2 < x < 2$

(d) $x \neq -1$ **(e)** $\cos x \leq 1 < 2$, $2 - \cos x > 0$, all x

5. **(a)** $x \leq 3$ **(b)** $-2 \leq x \leq 2$ **(c)** $x \geq 0$ **(d)** all x **(e)** all x

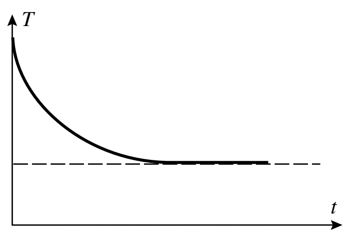
6. **(a)** $x \geq \frac{2}{3}$ **(b)** $-\frac{3}{2} \leq x \leq \frac{3}{2}$ **(c)** $x \geq 0$ **(d)** $x \neq 0$ **(e)** $x \geq 0$

7. **(a)** yes **(b)** yes
(c) no (vertical line test fails) **(d)** no (vertical line test fails)

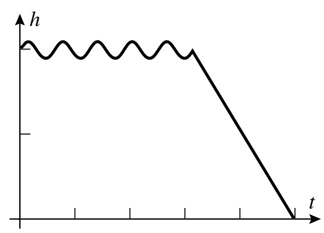
8. The sine of $\theta/2$ is $(L/2)/10$ (side opposite over hypotenuse), so that $L = 20\sin(\theta/2)$.

9. The cosine of θ is $(L - h)/L$ (side adjacent over hypotenuse), so $h = L(1 - \cos\theta)$.

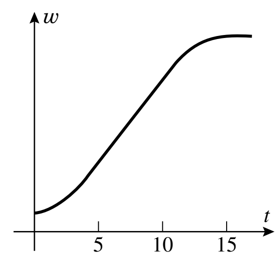
10.



11.



12.



13. **(a)** If $x < 0$, then $|x| = -x$ so $f(x) = -x + 3x + 1 = 2x + 1$. If $x \geq 0$, then $|x| = x$ so $f(x) = x + 3x + 1 = 4x + 1$;

$$f(x) = \begin{cases} 2x + 1, & x < 0 \\ 4x + 1, & x \geq 0 \end{cases}$$

(b) If $x < 0$, then $|x| = -x$ and $|x - 1| = 1 - x$ so $g(x) = -x + 1 - x = 1 - 2x$. If $0 \leq x < 1$, then $|x| = x$ and $|x - 1| = 1 - x$ so $g(x) = x + 1 - x = 1$. If $x \geq 1$, then $|x| = x$ and $|x - 1| = x - 1$ so $g(x) = x + x - 1 = 2x - 1$;

$$g(x) = \begin{cases} 1 - 2x, & x < 0 \\ 1, & 0 \leq x < 1 \\ 2x - 1, & x \geq 1 \end{cases}$$

14. **(a)** If $x < 5/2$, then $|2x - 5| = 5 - 2x$ so $f(x) = 3 + (5 - 2x) = 8 - 2x$. If $x \geq 5/2$, then $|2x - 5| = 2x - 5$ so $f(x) = 3 + (2x - 5) = 2x - 2$;

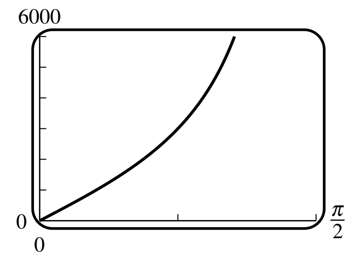
$$f(x) = \begin{cases} 8 - 2x, & x < 5/2 \\ 2x - 2, & x \geq 5/2 \end{cases}$$

(b) If $x < -1$, then $|x - 2| = 2 - x$ and $|x + 1| = -x - 1$ so $g(x) = 3(2 - x) - (-x - 1) = 7 - 2x$. If $-1 \leq x < 2$, then $|x - 2| = 2 - x$ and $|x + 1| = x + 1$ so $g(x) = 3(2 - x) - (x + 1) = 5 - 4x$. If $x \geq 2$, then $|x - 2| = x - 2$ and $|x + 1| = x + 1$ so $g(x) = 3(x - 2) - (x + 1) = 2x - 7$;

$$g(x) = \begin{cases} 7 - 2x, & x < -1 \\ 5 - 4x, & -1 \leq x < 2 \\ 2x - 7, & x \geq 2 \end{cases}$$

15. **(a)** $V = (8 - 2x)(15 - 2x)x$ **(b)** $-\infty < x < +\infty, -\infty < V < +\infty$ **(c)** $0 < x < 4$
(d) minimum value at $x = 0$ or at $x = 4$; maximum value somewhere in between (can be approximated by zooming with graphing calculator)

16. **(a)** $x = 3000 \tan \theta$ **(b)** $\theta \neq n\pi + \pi/2$ for n an integer, $-\infty < n < \infty$
(c) $0 \leq \theta < \pi/2$, $0 \leq x < +\infty$ **(d)** 3000 ft



17. **(i)** $x = 1, -2$ causes division by zero **(ii)** $g(x) = x + 1$, all x

18. **(i)** $x = 0$ causes division by zero **(ii)** $g(x) = \sqrt{x} + 1$ for $x \geq 0$

19. **(a)** 25°F **(b)** 2°F **(c)** −15°F

20. If $v = 48$ then $-60 = \text{WCI} = 1.6T - 55$; thus $T = (-60 + 55)/1.6 \approx -3$°F.

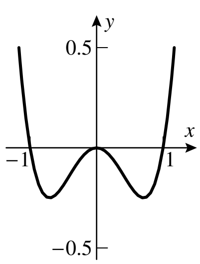
21. If $v = 8$ then $-10 = \text{WCI} = 91.4 + (91.4 - T)(0.0203(8) - 0.304\sqrt{8} - 0.474)$; thus $T = 91.4 + (10 + 91.4)/(0.0203(8) - 0.304\sqrt{8} - 0.474)$ and $T = 5$°F

22. The WCI is given by three formulae, but the first and third don't work with the data. Hence $-15 = \text{WCI} = 91.4 + (91.4 - 20)(0.0203v - 0.304\sqrt{v} - 0.474)$; set $x = \sqrt{v}$ so that $v = x^2$ and obtain $0.0203x^2 - 0.304x - 0.474 + (15 + 91.4)/(91.4 - 20) = 0$. Use the quadratic formula to find the two roots. Square them to get v and discard the spurious solution, leaving $v \approx 25$.

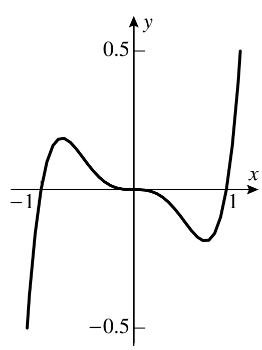
23. Let t denote time in minutes after 9:23 AM. Then $D(t) = 1000 - 20t$ ft.

EXERCISE SET 1.3

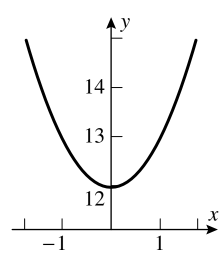
1. (e) seems best, though only (a) is bad.



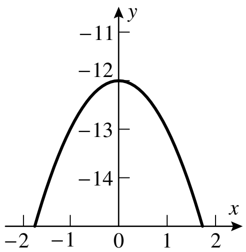
2. (e) seems best, though only (a) is bad and (b) is not good.



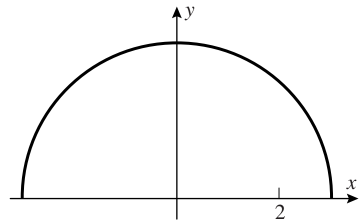
3. (b) and (c) are good; (a) is very bad.



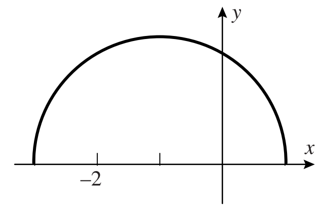
4. (b) and (c) are good;
(a) is very bad.



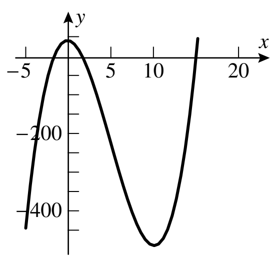
5. $[-3,3]\times[0,5]$



6. $[-4,2]\times[0,3]$

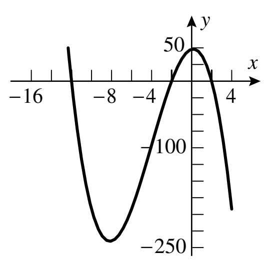


7. **(a)** window too narrow, too short
(b) window wide enough, but too short
(c) good window, good spacing
(d) window too narrow, too short



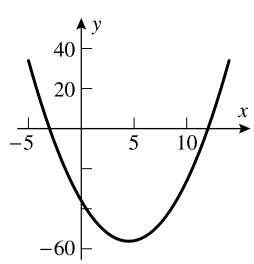
(e) window too narrow, too short

8. **(a)** window too narrow
(b) window too short
(c) good window, good tick spacing
(d) window too narrow, too short

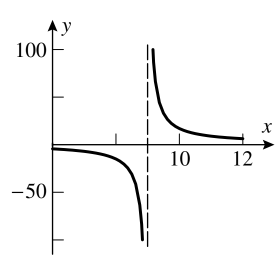


(e) shows one local minimum only, window too narrow, too short

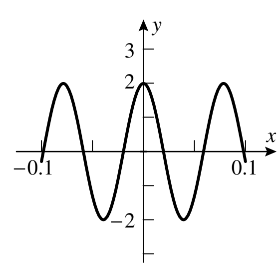
9. $[-5,14]\times[-60,40]$



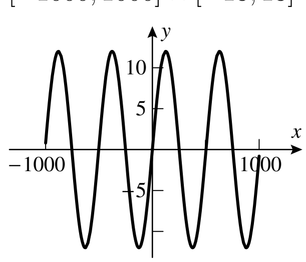
10. $[6,12]\times[-100,100]$



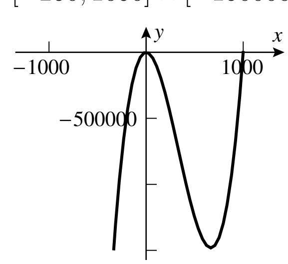
11. $[-0.1,0.1]\times[-3,3]$



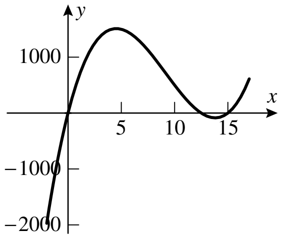
12. $[-1000,1000]\times[-13,13]$



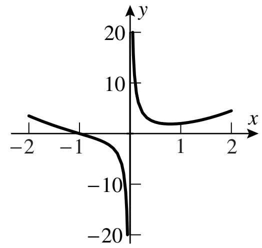
13. $[-250,1050]\times[-1500000,600000]$



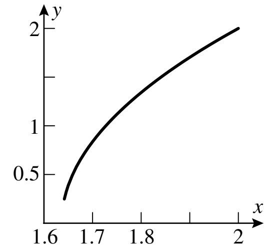
14. $[-3, 20] \times [-3500, 3000]$



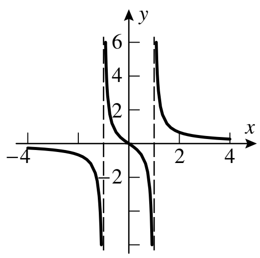
15. $[-2, 2] \times [-20, 20]$



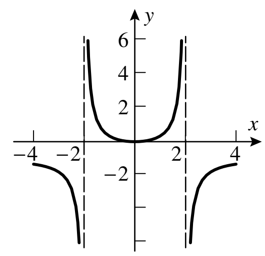
16. $[1.6, 2] \times [0, 2]$



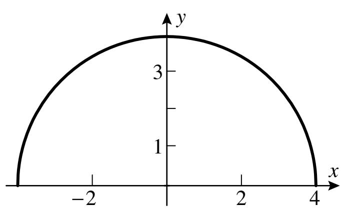
17. depends on graphing utility



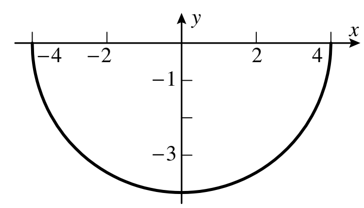
18. depends on graphing utility



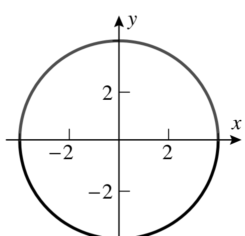
19. **(a)** $f(x) = \sqrt{16 - x^2}$



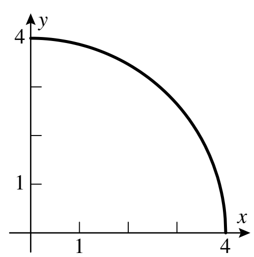
(b) $f(x) = -\sqrt{16 - x^2}$



(c)

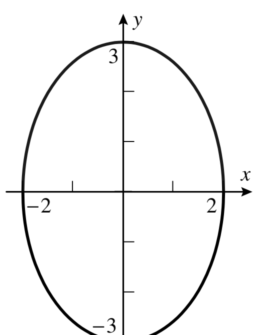


(d)

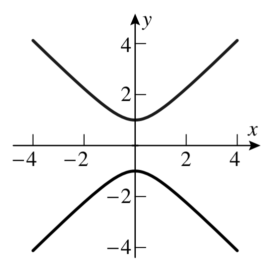


(e) No; the vertical line test fails.

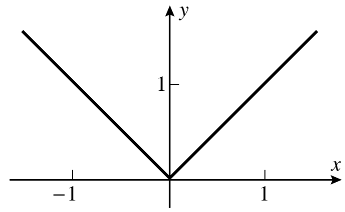
20. **(a)** $y = \pm 3\sqrt{1 - x^2/4}$



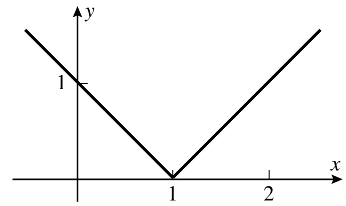
(b) $y = \pm\sqrt{x^2 + 1}$



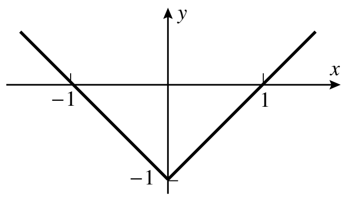
21. **(a)**



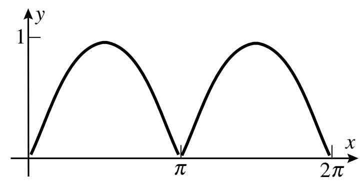
(b)



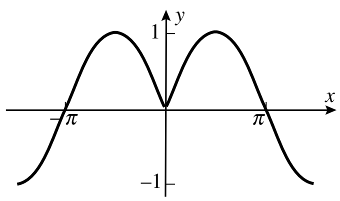
(c)



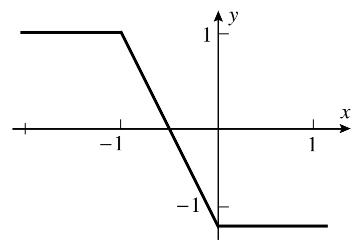
(d)



(e)



(f)



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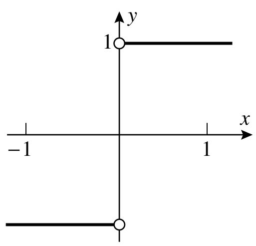
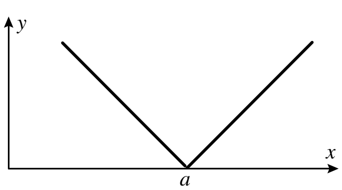


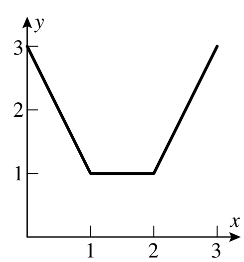
23. The portions of the graph of $y = f(x)$ which lie below the x-axis are reflected over the x-axis to give the graph of $y = |f(x)|$.

24. Erase the portion of the graph of $y = f(x)$ which lies in the left-half plane and replace it with the reflection over the y-axis of the portion in the right-half plane (symmetry over the y-axis) and you obtain the graph of $y = f(|x|)$.

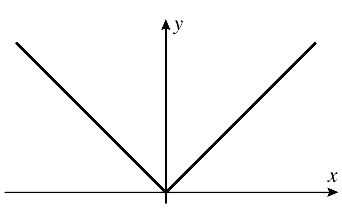
25. **(a)** for example, let $a = 1.1$



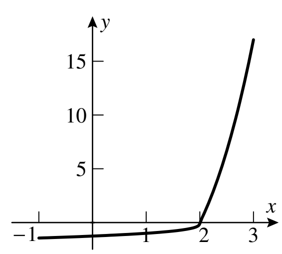
(b)



26. They are identical.

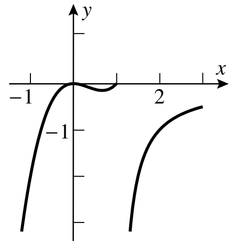


27.

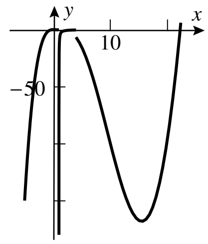


28. This graph is very complex. We show three views, small (near the origin), medium and large:

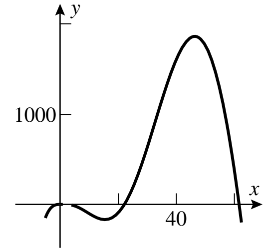
(a)



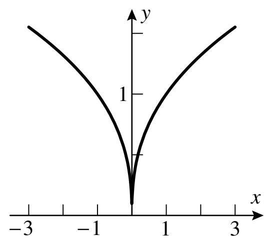
(b)



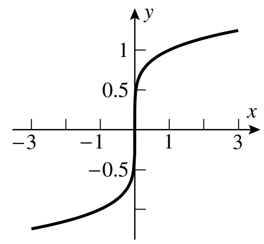
(c)



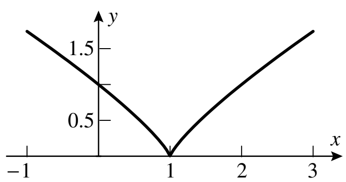
29. **(a)**



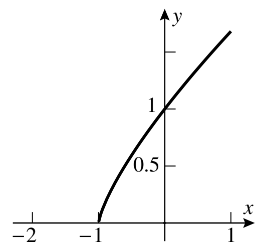
(b)



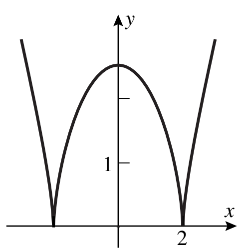
(c)



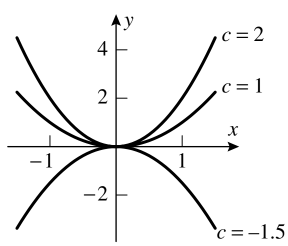
(d)



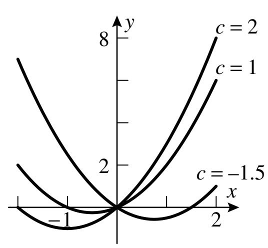
30.



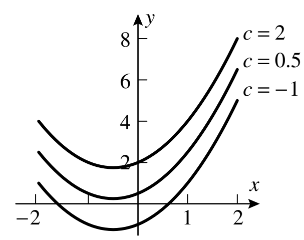
31. **(a)** stretches or shrinks the graph in the y-direction; reflects it over the x-axis if c changes sign



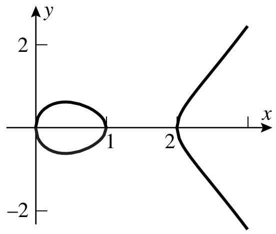
(b) As c increases, the parabola moves down and to the left. If c increases, up and right.



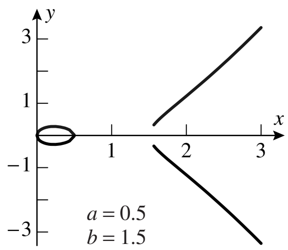
(c) The graph rises or falls in the y-direction with changes in c.

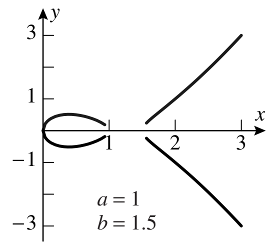


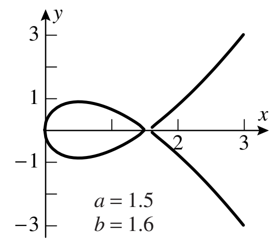
32. **(a)**



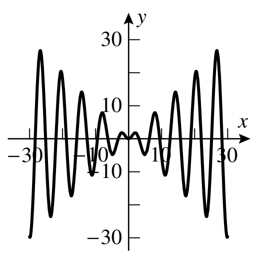
(b) x-intercepts at $x = 0, a, b$. Assume $a < b$ and let a approach b. The two branches of the curve come together. If a moves past b then a and b switch roles.



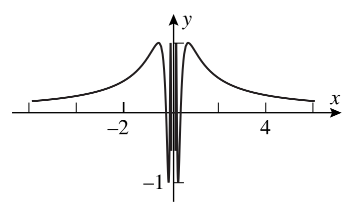




33. The curve oscillates between the lines $y = x$ and $y = -x$ with increasing rapidity as $|x|$ increases.

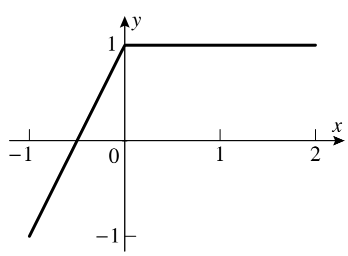


34. The curve oscillates between the lines $y = +1$ and $y = -1$, infinitely many times in any neighborhood of $x = 0$.

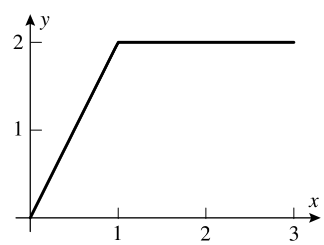


EXERCISE SET 1.4

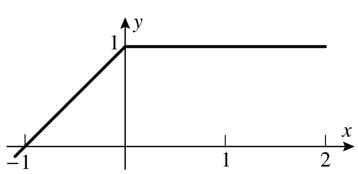
1. **(a)**



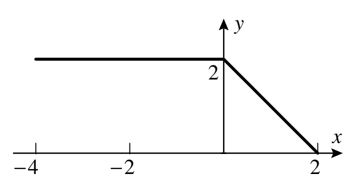
(b)



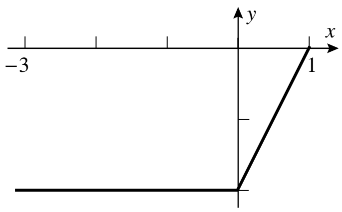
(c)



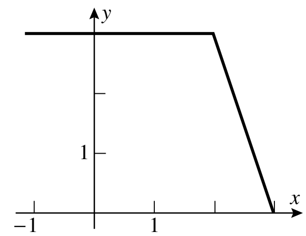
(d)



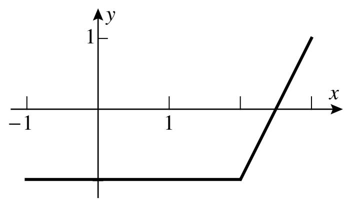
2. (a)



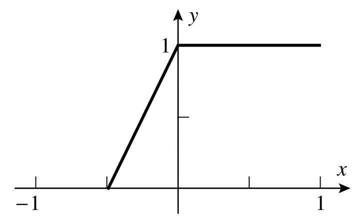
(b)



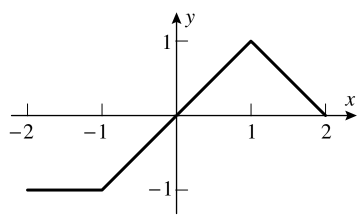
(c)



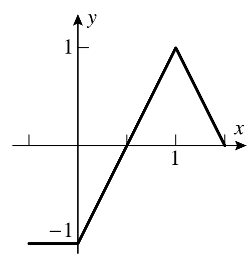
(d)



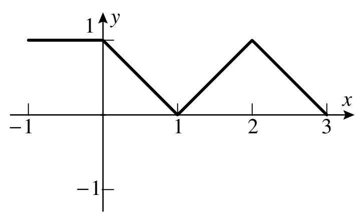
3. (a)



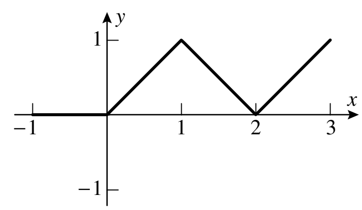
(b)



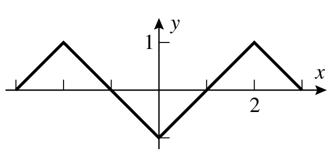
(c)



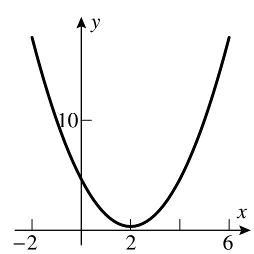
(d)



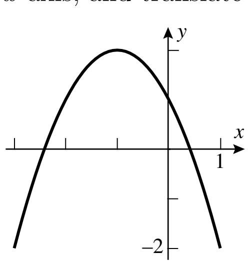
4.



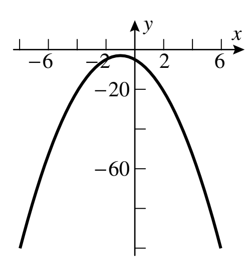
5. Translate right 2 units, and up one unit.



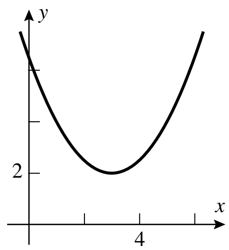
6. Translate left 1 unit, reflect over x-axis, and translate up 2 units.



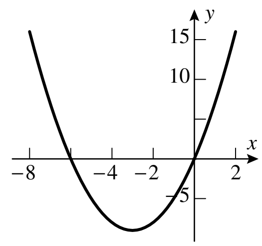
7. Translate left 1 unit, stretch vertically by a factor of 2, reflect over x-axis, translate down 3 units.



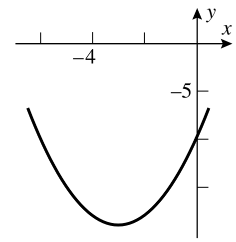
8. Translate right 3 units, compress vertically by a factor of $\frac{1}{2}$, and translate up 2 units.



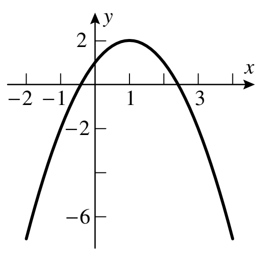
9. $y = (x+3)^2 - 9$; translate left 3 units and down 9 units.



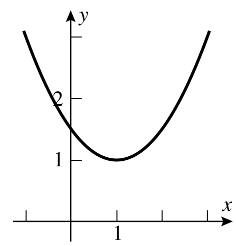
10. $y = (x+3)^2 - 19$; translate left 3 units and down 19 units.



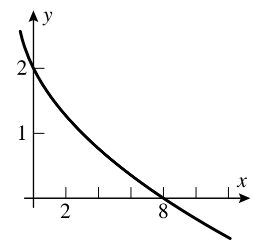
11. $y = -(x-1)^2 + 2$; translate right 1 unit, reflect over x-axis, translate up 2 units.



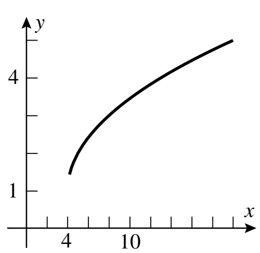
12. $y = \frac{1}{2}[(x-1)^2 + 2]$; translate left 1 unit and up 2 units, compress vertically by a factor of $\frac{1}{2}$.



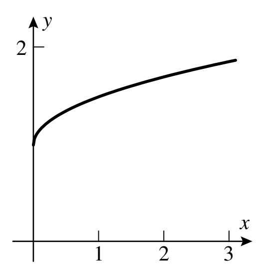
13. Translate left 1 unit, reflect over x-axis, translate up 3 units.



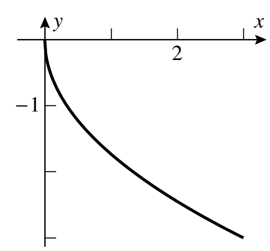
14. Translate right 4 units and up 1 unit.



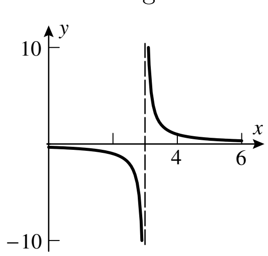
15. Compress vertically by a factor of $\frac{1}{2}$, translate up 1 unit.



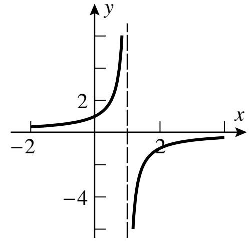
16. Stretch vertically by a factor of $\sqrt{3}$ and reflect over x-axis.



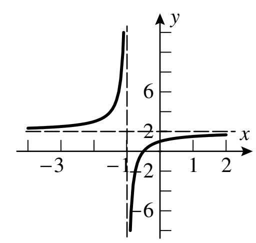
17. Translate right 3 units.



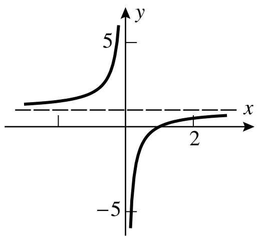
18. Translate right 1 unit and reflect over x-axis.



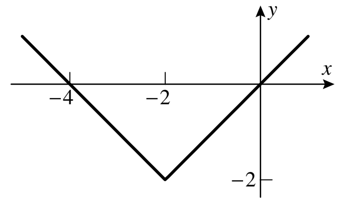
19. Translate left 1 unit, reflect over x-axis, translate up 2 units.



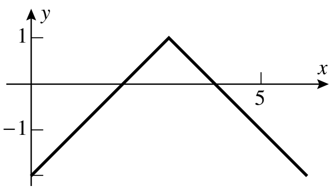
20. $y = 1 - 1/x$; reflect over x-axis, translate up 1 unit.



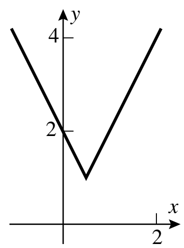
21. Translate left 2 units and down 2 units.



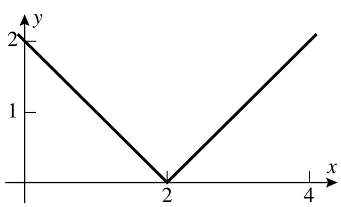
22. Translate right 3 units, reflect over x-axis, translate up 1 unit.



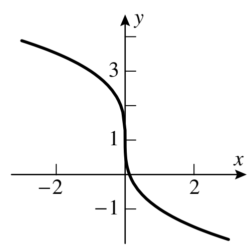
23. Stretch vertically by a factor of 2, translate right 1 unit and up 1 unit.



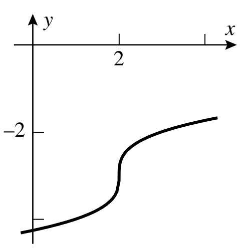
24. $y = |x - 2|$; translate right 2 units.



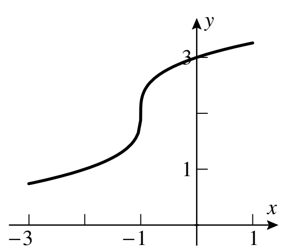
25. Stretch vertically by a factor of 2, reflect over x-axis, translate up 1 unit.



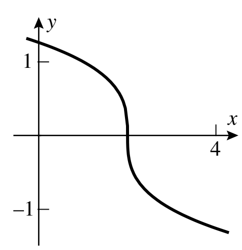
26. Translate right 2 units and down 3 units.



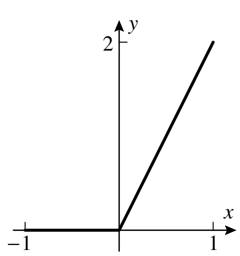
27. Translate left 1 unit and up 2 units.



28. Translate right 2 units, reflect over x-axis.

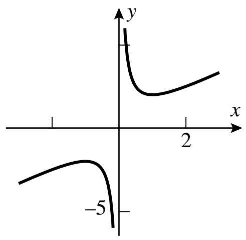


29. **(a)**



(b) $y = \begin{cases} 0 \text{ if } x \le 0 \\ 2x \text{ if } 0 < x \end{cases}$

30.



31. $(f+g)(x) = x^2+2x+1$, all x; $(f-g)(x) = 2x-x^2-1$, all x; $(fg)(x) = 2x^3+2x$, all x; $(f/g)(x) = 2x/(x^2+1)$, all x

32. $(f+g)(x) = 3x-2+|x|$, all x; $(f-g)(x) = 3x-2-|x|$, all x; $(fg)(x) = 3x|x|-2|x|$, all x; $(f/g)(x) = (3x-2)/|x|$, all $x \neq 0$

33. $(f+g)(x) = 3\sqrt{x-1}$, $x \geq 1$; $(f-g)(x) = \sqrt{x-1}$, $x \geq 1$; $(fg)(x) = 2x-2$, $x \geq 1$; $(f/g)(x) = 2, x > 1$

34. $(f+g)(x) = (2x^2+1)/[x(x^2+1)]$, all $x \neq 0$; $(f-g)(x) = -1/[x(x^2+1)]$, all $x \neq 0$; $(fg)(x) = 1/(x^2+1)$, all $x \neq 0$; $(f/g)(x) = x^2/(x^2+1)$, all $x \neq 0$

35. **(a)** 3 **(b)** 9 **(c)** 2 **(d)** 2

36. **(a)** $\pi - 1$ **(b)** 0 **(c)** $-\pi^2+3\pi-1$ **(d)** 1

37. **(a)** t^4+1 **(b)** t^2+4t+5 **(c)** x^2+4x+5 **(d)** $\dfrac{1}{x^2}+1$
(e) $x^2+2xh+h^2+1$ **(f)** x^2+1 **(g)** $x+1$ **(h)** $9x^2+1$

38. **(a)** $\sqrt{5s+2}$ **(b)** $\sqrt{\sqrt{x}+2}$ **(c)** $3\sqrt{5x}$ **(d)** $1/\sqrt{x}$
(e) $\sqrt[4]{x}$ **(f)** 0 **(g)** $1/\sqrt[4]{x}$ **(h)** $|x-1|$

39. $(f \circ g)(x) = 2x^2-2x+1$, all x; $(g \circ f)(x) = 4x^2+2x$, all x

40. $(f \circ g)(x) = 2-x^6$, all x; $(g \circ f)(x) = -x^6+6x^4-12x^2+8$, all x

41. $(f \circ g)(x) = 1-x$, $x \leq 1$; $(g \circ f)(x) = \sqrt{1-x^2}$, $|x| \leq 1$

42. $(f \circ g)(x) = \sqrt{\sqrt{x^2+3}-3}$, $|x| \geq \sqrt{6}$; $(g \circ f)(x) = \sqrt{x}$, $x \geq 3$

43. $(f \circ g)(x) = \dfrac{1}{1-2x}$, $x \neq \dfrac{1}{2}, 1$; $(g \circ f)(x) = -\dfrac{1}{2x} - \dfrac{1}{2}$, $x \neq 0, 1$

44. $(f \circ g)(x) = \dfrac{x}{x^2+1}$, $x \neq 0$; $(g \circ f)(x) = \dfrac{1}{x} + x$, $x \neq 0$

45. $x^{-6}+1$

46. $\dfrac{x}{x+1}$

47. **(a)** $g(x) = \sqrt{x}$, $h(x) = x+2$ **(b)** $g(x) = |x|$, $h(x) = x^2-3x+5$

48. **(a)** $g(x) = x+1$, $h(x) = x^2$ **(b)** $g(x) = 1/x$, $h(x) = x-3$

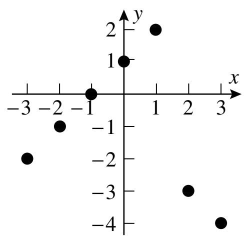
49. **(a)** $g(x) = x^2$, $h(x) = \sin x$ **(b)** $g(x) = 3/x$, $h(x) = 5+\cos x$

50. **(a)** $g(x) = 3\sin x,\ h(x) = x^2$ **(b)** $g(x) = 3x^2 + 4x,\ h(x) = \sin x$

51. **(a)** $f(x) = x^3,\ g(x) = 1 + \sin x,\ h(x) = x^2$ **(b)** $f(x) = \sqrt{x},\ g(x) = 1 - x,\ h(x) = \sqrt[3]{x}$

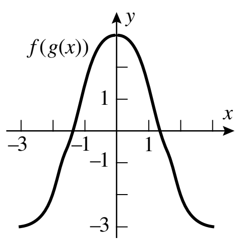
52. **(a)** $f(x) = 1/x,\ g(x) = 1 - x,\ h(x) = x^2$ **(b)** $f(x) = |x|,\ g(x) = 5 + x,\ h(x) = 2x$

53.

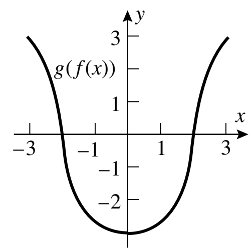


54. $\{-2, -1, 0, 1, 2, 3\}$

55. Note that
$f(g(-x)) = f(-g(x)) = f(g(x))$,
so $f(g(x))$ is even.



56. Note that $g(f(-x)) = g(f(x))$,
so $g(f(x))$ is even.



57. $f(g(x)) = 0$ when $g(x) = \pm 2$, so $x = \pm 1.4$; $g(f(x)) = 0$ when $f(x) = 0$, so $x = \pm 2$.

58. $f(g(x)) = 0$ at $x = -1$ and $g(f(x)) = 0$ at $x = -1$

59. $$\frac{3(x+h)^2 - 5 - (3x^2 - 5)}{h} = \frac{6xh + 3h^2}{h} = 6x + 3h;$$

$$\frac{3w^2 - 5 - (3x^2 - 5)}{w - x} = \frac{3(w-x)(w+x)}{w - x} = 3w + 3x$$

60. $$\frac{(x+h)^2 + 6(x+h) - (x^2 + 6x)}{h} = \frac{2xh + h^2 + 6h}{h} = 2x + h + 6;$$

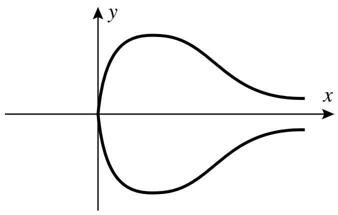
$$\frac{w^2 + 6w - (x^2 + 6x)}{w - x} = w + x + 6$$

61. $$\frac{1/(x+h) - 1/x}{h} = \frac{x - (x+h)}{xh(x+h)} = \frac{-1}{x(x+h)};\ \frac{1/w - 1/x}{w - x} = \frac{x - w}{wx(w-x)} = -\frac{1}{xw}$$

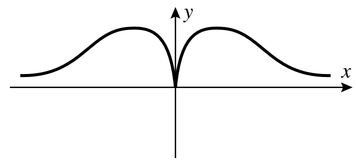
62. $$\frac{1/(x+h)^2 - 1/x^2}{h} = \frac{x^2 - (x+h)^2}{x^2h(x+h)^2} = -\frac{2x+h}{x^2(x+h)^2};\ \frac{1/w^2 - 1/x^2}{w - x} = \frac{x^2 - w^2}{x^2w^2(w-x)} = -\frac{x+w}{x^2w^2}$$

63. **(a)** the origin **(b)** the x-axis **(c)** the y-axis **(d)** none

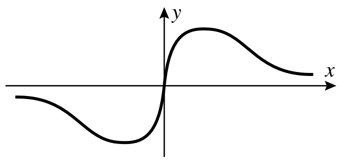
64. **(a)**



(b)



(c)



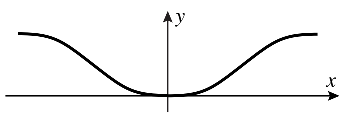
65. **(a)**

x	-3	-2	-1	0	1	2	3
$f(x)$	1	-5	-1	0	-1	-5	1

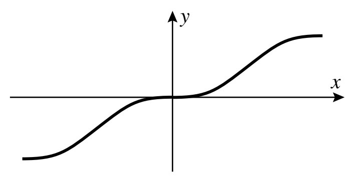
(b)

x	-3	-2	-1	0	1	2	3
$f(x)$	1	5	-1	0	1	-5	-1

66. **(a)**



(b)



67. **(a)** even **(b)** odd **(c)** odd **(d)** neither

68. neither; odd; even

69. **(a)** $f(-x) = (-x)^2 = x^2 = f(x)$, even **(b)** $f(-x) = (-x)^3 = -x^3 = -f(x)$, odd

(c) $f(-x) = |-x| = |x| = f(x)$, even **(d)** $f(-x) = -x + 1$, neither

(e) $f(-x) = \dfrac{(-x)^3 - (-x)}{1 + (-x)^2} = -\dfrac{x^3 + x}{1 + x^2} = -f(x)$, odd

(f) $f(-x) = 2 = f(x)$, even

70. **(a)** x-axis, because $x = 5(-y)^2 + 9$ gives $x = 5y^2 + 9$

(b) x-axis, y-axis, and origin, because $x^2 - 2(-y)^2 = 3$, $(-x)^2 - 2y^2 = 3$, and $(-x)^2 - 2(-y)^2 = 3$ all give $x^2 - 2y^2 = 3$

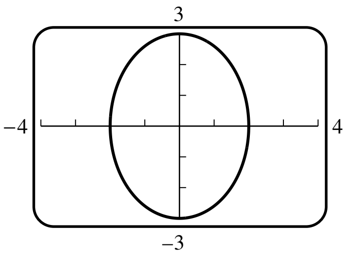
(c) origin, because $(-x)(-y) = 5$ gives $xy = 5$

71. **(a)** y-axis, because $(-x)^4 = 2y^3 + y$ gives $x^4 = 2y^3 + y$

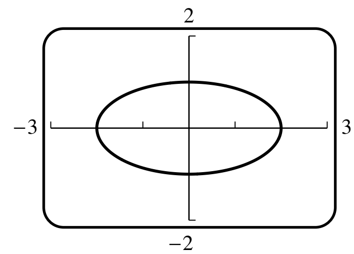
(b) origin, because $(-y) = \dfrac{(-x)}{3 + (-x)^2}$ gives $y = \dfrac{x}{3 + x^2}$

(c) x-axis, y-axis, and origin because $(-y)^2 = |x| - 5$, $y^2 = |-x| - 5$, and $(-y)^2 = |-x| - 5$ all give $y^2 = |x| - 5$

72.



73.

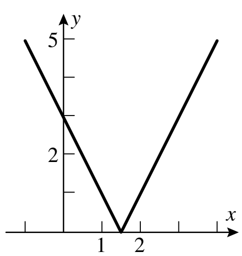


74. **(a)** Whether we replace x with $-x$, y with $-y$, or both, we obtain the same equation, so by Theorem 1.4.3 the graph is symmetric about the x-axis, the y-axis and the origin.

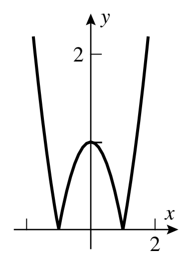
(b) $y = (1 - x^{2/3})^{3/2}$

(c) For quadrant II, the same; for III and IV use $y = -(1 - x^{2/3})^{3/2}$. (For graphing it may be helpful to use the tricks that precede Exercise 29 in Section 1.3.)

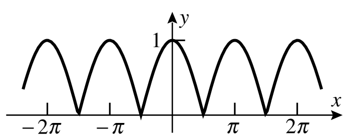
75.



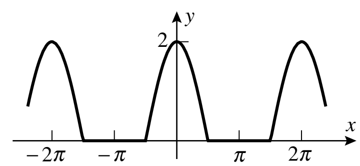
76.



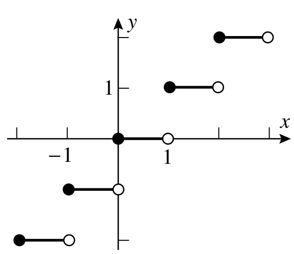
77. **(a)**



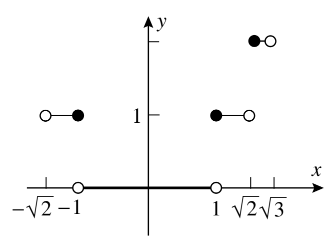
(b)



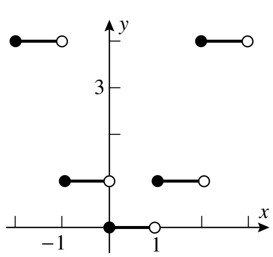
78. **(a)**



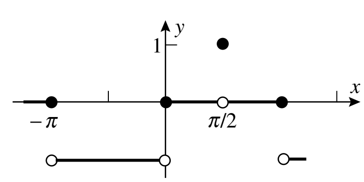
(b)



(c)



(d)



79. Yes, e.g. $f(x) = x^k$ and $g(x) = x^n$ where k and n are integers.

80. If $x \geq 0$ then $|x| = x$ and $f(x) = g(x)$. If $x < 0$ then $f(x) = |x|^{p/q}$ if p is even and $f(x) = -|x|^{p/q}$ if p is odd; in both cases $f(x)$ agrees with $g(x)$.

EXERCISE SET 1.5

1. **(a)** $\dfrac{3-0}{0-2}=-\dfrac{3}{2}, \dfrac{3-(8/3)}{0-6}=-\dfrac{1}{18}, \dfrac{0-(8/3)}{2-6}=\dfrac{2}{3}$

(b) Yes; the first and third slopes above are negative reciprocals of each other.

2. **(a)** $\dfrac{-1-(-1)}{-3-5}=0, \dfrac{-1-3}{5-7}=2, \dfrac{3-3}{7-(-1)}=0, \dfrac{-1-3}{-3-(-1)}=2$

(b) Yes; there are two pairs of equal slopes, so two pairs of parallel lines.

3. III < II < IV < I

4. III < IV < I < II

5. **(a)** $\dfrac{1-(-5)}{1-(-2)}=2, \dfrac{-5-(-1)}{-2-0}=2, \dfrac{1-(-1)}{1-0}=2$. Since the slopes connecting all pairs of points are equal, they lie on a line.

(b) $\dfrac{4-2}{-2-0}=-1, \dfrac{2-5}{0-1}=3, \dfrac{4-5}{-2-1}=\dfrac{1}{3}$. Since the slopes connecting the pairs of points are not equal, the points do not lie on a line.

6. The slope, $m=-2$, is obtained from $\dfrac{y-5}{x-7}$, and thus $y-5=-2(x-7)$.

(a) If $x=9$ then $y=1$.

(b) If $y=12$ then $x=7/2$.

7. The slope, $m=3$, is equal to $\dfrac{y-2}{x-1}$, and thus $y-2=3(x-1)$.

(a) If $x=5$ then $y=14$.

(b) If $y=-2$ then $x=-1/3$.

8. **(a)** Compute the slopes: $\dfrac{y-0}{x-0}=\dfrac{1}{2}$ or $y=x/2$. Also $\dfrac{y-5}{x-7}=2$ or $y=2x-9$. Solve simultaneously to obtain $x=6, y=3$.

9. **(a)** The first slope is $\dfrac{2-0}{1-x}$ and the second is $\dfrac{5-0}{4-x}$. Since they are negatives of each other we get $2(4-x)=-5(1-x)$ or $7x=13$, $x=13/7$.

10. **(a)** 27° **(b)** 135° **(c)** 63° **(d)** 91°

11. **(a)** 153° **(b)** 45° **(c)** 117° **(d)** 89°

12. **(a)** $m=\tan\phi=-\sqrt{3}/3$, so $\phi=150°$

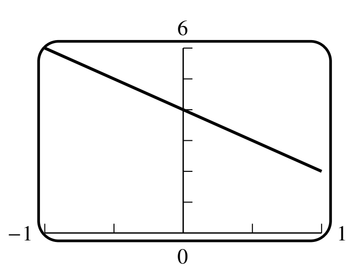
(b) $m=\tan\phi=4$, so $\phi=76°$

13. **(a)** $m=\tan\phi=\sqrt{3}$, so $\phi=60°$

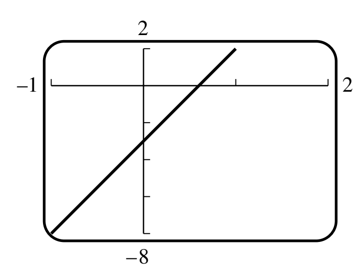
(b) $m=\tan\phi=-2$, so $\phi=117°$

14. $y=0$ and $x=0$ respectively

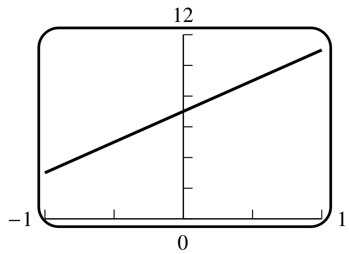
15. $y=-2x+4$



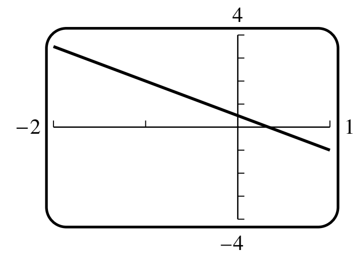
16. $y=5x-3$



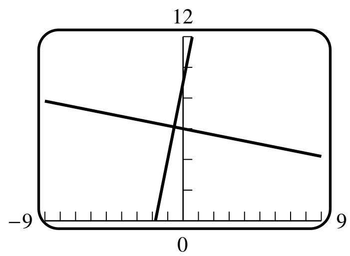
17. Parallel means the lines have equal slopes, so $y = 4x + 7$.



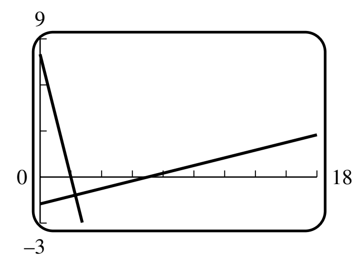
18. The slope of both lines is $-3/2$, so $y - 2 = (-3/2)(x - (-1))$, or $y = -\frac{3}{2}x + \frac{1}{2}$



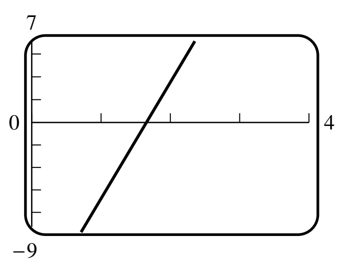
19. The negative reciprocal of 5 is $-1/5$, so $y = -\frac{1}{5}x + 6$.



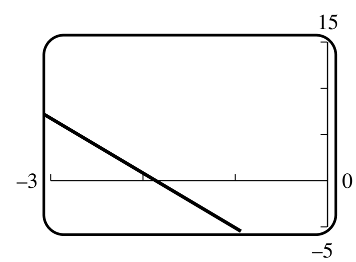
20. The slope of $x - 4y = 7$ is $1/4$ whose negative reciprocal is -4, so $y - (-4) = -4(x - 3)$ or $y = -4x + 8$.



21. $m = \dfrac{4 - (4 - 7)}{2 - 1} = 11$, so $y - (-7) = 11(x - 1)$, or $y = 11x - 18$



22. $m = \dfrac{6 - 1}{-3 - (-2)} = -5$, so $y - 6 = -5(x - (-3))$, or $y = -5x - 9$



23. **(a)** $m_1 = m_2 = 4$, parallel
(b) $m_1 = 2 = -1/m_2$, perpendicular
(c) $m_1 = m_2 = 5/3$, parallel
(d) If $A \neq 0$ and $B \neq 0$ then $m_1 = -A/B = -1/m_2$, perpendicular; if $A = 0$ or $B = 0$ (not both) then one line is horizontal, the other vertical, so perpendicular.
(e) neither

24. **(a)** $m_1 = m_2 = -5$, parallel
(b) $m_1 = 2 = -1/m_2$, perpendicular
(c) $m_1 = -4/5 = -1/m_2$, perpendicular
(d) If $B \neq 0$, $m_1 = m_2 = -A/B$; if $B = 0$ both are vertical, so parallel
(e) neither

25. **(a)** $m = (0 - (-3))/(2 - 0)) = 3/2$ so $y = 3x/2 - 3$
(b) $m = (-3 - 0)/(4 - 0) = -3/4$ so $y = -3x/4$

26. **(a)** $m = (0 - 2)/(2 - 0)) = -1$ so $y = -x + 2$
(b) $m = (2 - 0)/(3 - 0) = 2/3$ so $y = 2x/3$

27. **(a)** The velocity is the slope, which is $\dfrac{5-(-4)}{10-0} = 9/10$ ft/s.

(b) $x = -4$

(c) The line has slope 9/10 and passes through $(0, -4)$, so has equation $x = 9t/10 - 4$; at $t = 2$, $x = -2.2$.

(d) $t = 80/9$

28. **(a)** $v = \dfrac{5-1}{4-2} = 2$ m/s **(b)** $x - 1 = 2(t-2)$ or $x = 2t - 3$ **(c)** $x = -3$

29. **(a)** The acceleration is the slope of the velocity, so $a = \dfrac{3-(-1)}{1-4} = -\dfrac{4}{3}$ ft/s^2.

(b) $v - 3 = -\frac{4}{3}(t-1)$, or $v = -\frac{4}{3}t + \frac{13}{3}$ **(c)** $v = \frac{13}{3}$ ft/s

30. **(a)** The acceleration is the slope of the velocity, so $a = \dfrac{0-5}{10-0} = -\dfrac{5}{10} = -\dfrac{1}{2}$ ft/s^2.

(b) $v = 5$ ft/s **(c)** $v = 4$ ft/s **(d)** $t = 4$ s

31. **(a)** It moves (to the left) 6 units with velocity $v = -3$ cm/s, then remains motionless for 5 s, then moves 3 units to the left with velocity $v = -1$ cm/s.

(b) $v_{\text{ave}} = \dfrac{0-9}{10-0} = -\dfrac{9}{10}$ cm/s

(c) Since the motion is in one direction only, the speed is the negative of the velocity, so $s_{\text{ave}} = \frac{9}{10}$ cm/s.

32. It moves right with constant velocity $v = 5$ km/h; then accelerates; then moves with constant, though increased, velocity again; then slows down.

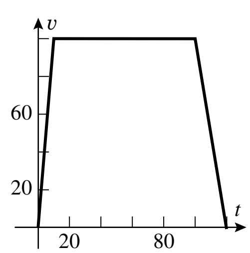
33. **(a)** If x_1 denotes the final position and x_0 the initial position, then $v = (x_1 - x_0)/(t_1 - t_0) = 0$ mi/h, since $x_1 = x_0$.

(b) If the distance traveled in one direction is d, then the outward journey took $t = d/40$ h. Thus $s_{\text{ave}} = \dfrac{\text{total dist}}{\text{total time}} = \dfrac{2d}{t + (2/3)t} = \dfrac{80t}{t + (2/3)t} = 48$ mi/h.

(c) $t + (2/3)t = 5$, so $t = 3$ and $2d = 80t = 240$ mi round trip

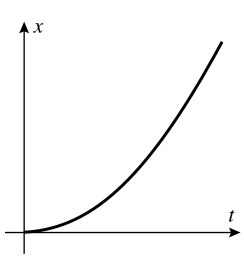
34. **(a)** down, since $v < 0$ **(b)** $v = 0$ at $t = 2$ **(c)** It's constant at 32 ft/s^2.

35. **(a)**



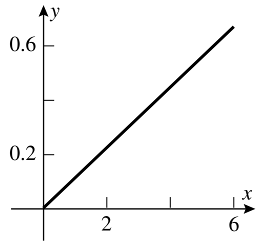
(b) $v = \begin{cases} 10t & \text{if } 0 \le t \le 10 \\ 100 & \text{if } 10 \le t \le 100 \\ 600 - 5t & \text{if } 100 \le t \le 120 \end{cases}$

36.



37. **(a)** $y = 20 - 15 = 5$ when $x = 45$, so $5 = 45k$, $k = 1/9$, $y = x/9$

(b)

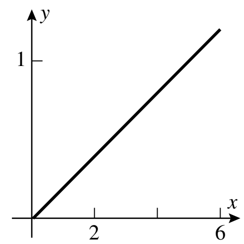


(c) $l = 15 + y = 15 + 100(1/9) = 26.11$ in.

(d) If $y_{\max} = 15$ then solve $15 = kx = x/9$ for $x = 135$ lb.

38. **(a)** Since $y = 0.2 = (1)k$, $k = 1/5$ and $y = x/5$

(b)



(c) $y = 3k = 3/5$ so 0.6 ft.

(d) $y_{\max} = (1/2)3 = 1.5$ so solve $1.5 = x/5$ for $x = 7.5$ tons

39. Each increment of 1 in the value of x yields the increment of 1.2 for y, so the relationship is linear. If $y = mx + b$ then $m = 1.2$; from $x = 0$, $y = 2$, follows $b = 2$, so $y = 1.2x + 2$

40. Each increment of 1 in the value of x yields the increment of -2.1 for y, so the relationship is linear. If $y = mx + b$ then $m = -2.1$; from $x = 0$, $y = 10.5$ follows $b = 10.5$, so $y = -2.1x + 10.5$

41. **(a)** With T_F as independent variable, we have $\dfrac{T_C - 100}{T_F - 212} = \dfrac{0 - 100}{32 - 212}$, so $T_C = \dfrac{5}{9}(T_F - 32)$.

(b) 5/9

(c) Set $T_F = T_C = \frac{5}{9}(T_F - 32)$ and solve for T_F: $T_F = T_C = -40°$ (F or C).

(d) 37° C

42. **(a)** One degree Celsius is one degree Kelvin, so the slope is the ratio $1/1 = 1$.
Thus $T_C = T_K - 273.15$.

(b) $T_C = 0 - 273.15 = -273.15°$ C

43. **(a)** $\dfrac{p - 1}{h - 0} = \dfrac{5.9 - 1}{50 - 0}$, or $p = 0.098h + 1$ **(b)** when $p = 2$, or $h = 1/0.098 \approx 10.20$ m

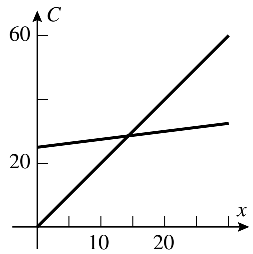
44. **(a)** $\dfrac{R - 123.4}{T - 20} = \dfrac{133.9 - 123.4}{45 - 20}$, so $R = 0.42T + 115$. **(b)** $T = 32.38°$C

45. **(a)** $\dfrac{r - 0.80}{t - 0} = \dfrac{0.75 - 0.80}{4 - 0}$, so $r = -0.0125t + 0.8$ **(b)** 64 days

46. **(a)** Let the position at rest be y_0. Then $y_0 + y = y_0 + kx$; with $x = 11$ we get $y_0 + kx = y_0 + 11k = 40$, and with $x = 24$ we get $y_0 + kx = y_0 + 24k = 60$. Solve to get $k = 20/13$ and $y_0 = 300/13$.

(b) $300/13 + (20/13)W = 30$, so $W = (390 - 300)/20 = 9/2$ g.

47. **(a)** For x trips we have $C_1 = 2x$ and $C_2 = 25 + x/4$

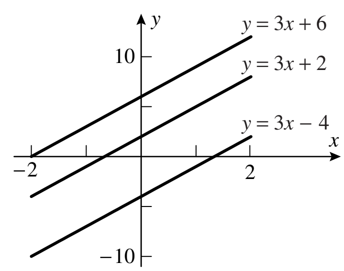


(b) $2x = 25 + x/4$, or $x = 100/7$, so the commuter pass becomes worthwhile at $x = 15$.

48. If the student drives x miles, then the total costs would be $C_A = 4000 + (1.25/20)x$ and $C_B = 5500 + (1.25/30)x$. Set $4000 + 5x/80 = 5500 + 5x/120$ and solve for $x = 72,000$ mi.

EXERCISE SET 1.6

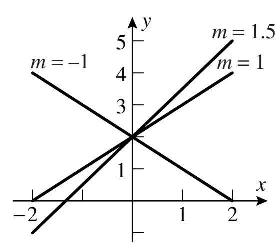
1. **(a)** $y = 3x + b$ **(b)** $y = 3x + 6$ **(c)**



2. Since the slopes are negative reciprocals, $y = -\frac{1}{3}x + b$.

3. **(a)** $y = mx + 2$

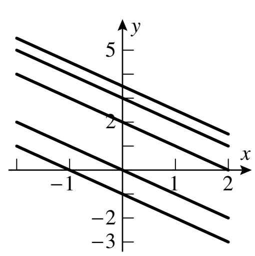
(b) $m = \tan\phi = \tan 135° = -1$, so $y = -x + 2$ **(c)**



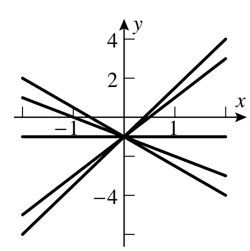
4. **(a)** $y = mx$ **(b)** $y = m(x - 1)$

(c) $y = -2 + m(x - 1)$ **(d)** $2x + 4y = C$

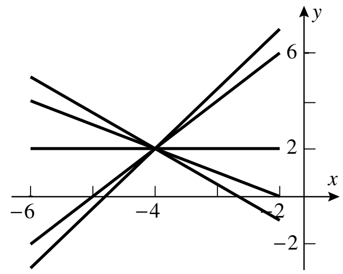
5. **(a)** The slope is -1.



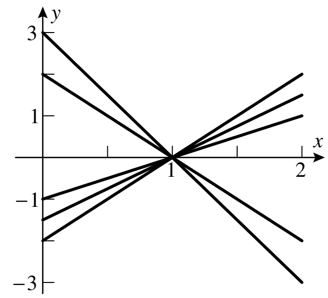
(b) The y-intercept is $y = -1$.



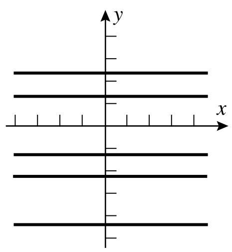
(c) They pass through the point $(-4, 2)$.



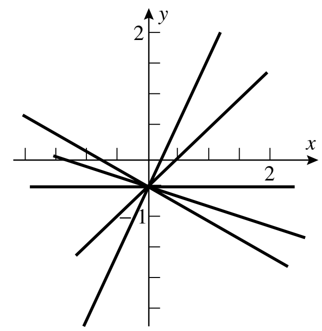
(d) The x-intercept is $x = 1$.



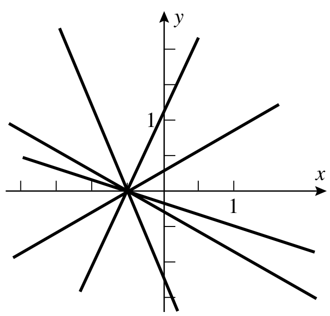
6. **(a)** horizontal lines



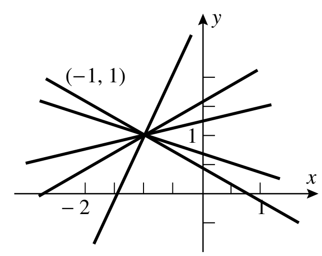
(b) The y-intercept is $y = -1/2$.



(c) The x-intercept is $x = -1/2$.



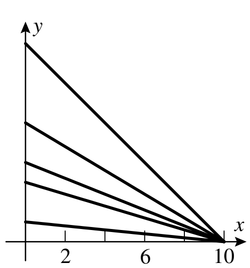
(d) They pass through $(-1, 1)$.



7. Let the line be tangent to the circle at the point (x_0, y_0) where $x_0^2 + y_0^2 = 9$. The slope of the tangent line is the negative reciprocal of y_0/x_0 (why?), so $m = -x_0/y_0$ and $y = -(x_0/y_0)x + b$. Substituting the point (x_0, y_0) as well as $y_0 = \pm\sqrt{9 - x_0^2}$ we get $y = \pm\dfrac{9 - x_0 x}{\sqrt{9 - x_0^2}}$.

8. Solve the simultaneous equations to get the point $(-2, 1/3)$ of intersection. Then $y = \frac{1}{3} + m(x + 2)$.

9. The x-intercept is $x = 10$ so that with depreciation at 10% per year the final value is always zero, and hence $y = m(x - 10)$. The y-intercept is the original value.

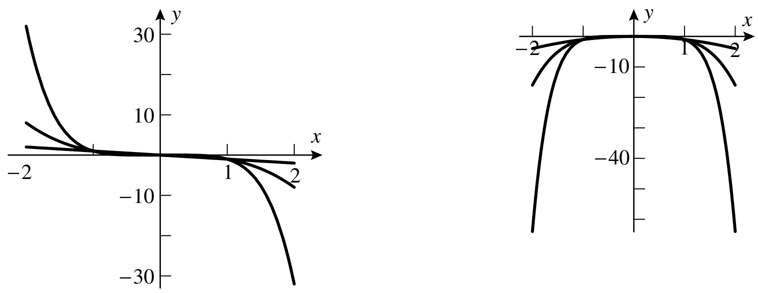


10. A line through $(6,-1)$ has the form $y+1=m(x-6)$. The intercepts are $x=6+1/m$ and $y=-6m-1$. Set $-(6+1/m)(6m+1)=3$, or $36m^2+15m+1=(12m+1)(3m+1)=0$ with roots $m=-1/12,-1/3$; thus $y+1=-(1/3)(x-6)$ and $y+1=-(1/12)(x-6)$.

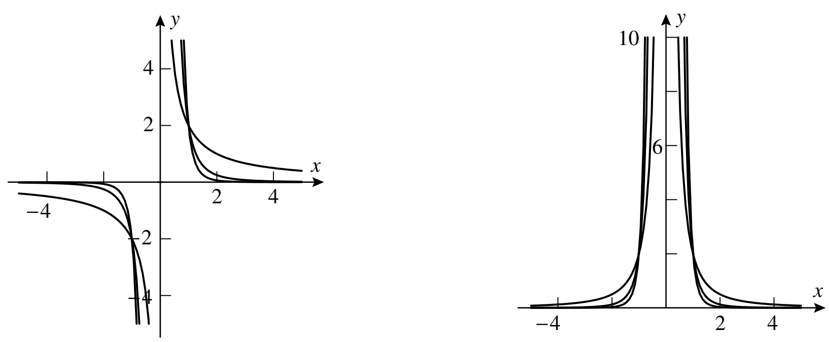
11. **(a)** VI **(b)** IV **(c)** III **(d)** V **(e)** I **(f)** II

12. In all cases k must be positive, or negative values would appear in the chart. Only kx^{-3} decreases, so that must be $f(x)$. Next, kx^2 grows faster than $kx^{3/2}$, so that would be $g(x)$, which grows faster than $h(x)$ (to see this, consider ratios of successive values of the functions). Finally, experimentation (a spreadsheet is handy) for values of k yields (approximately) $f(x)=10x^{-3}$, $g(x)=x^2/2$, $h(x)=2x^{1.5}$.

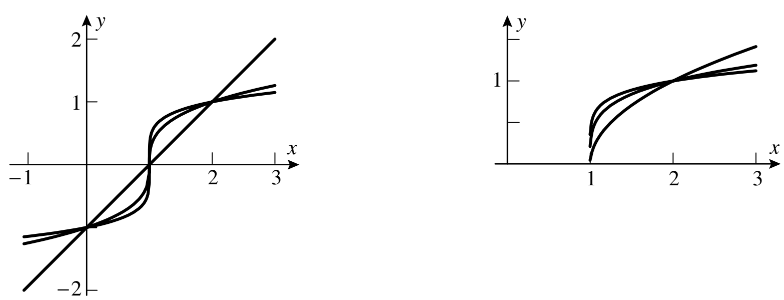
13. **(a)**



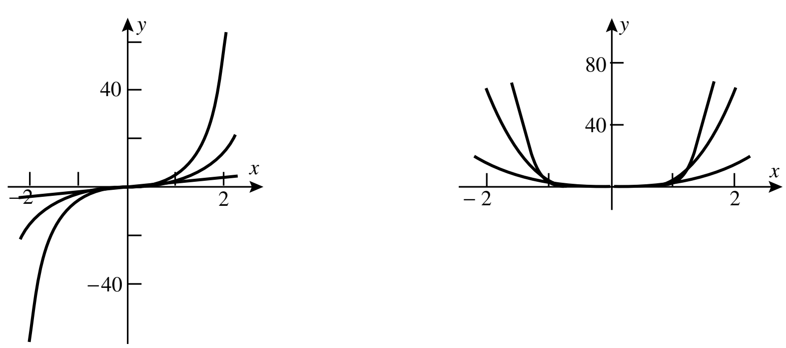
(b)



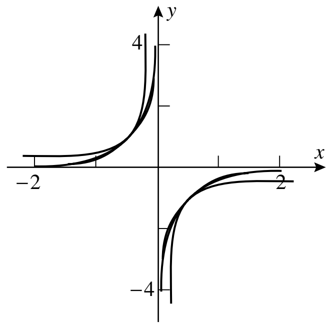
(c)

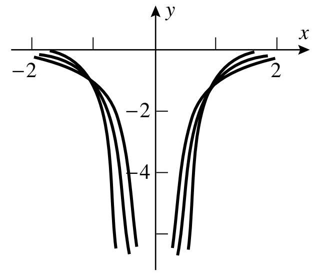


14. **(a)**



(b)





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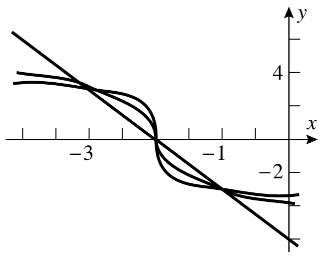
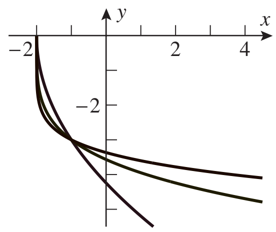
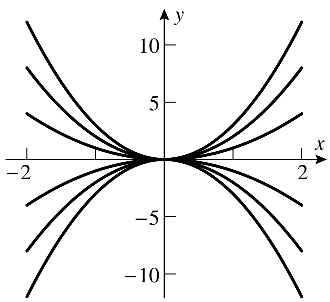
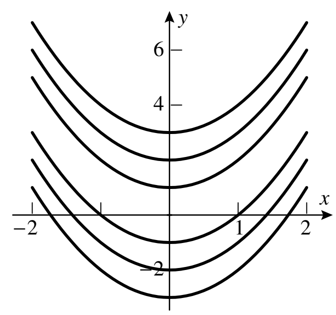




15. **(a)**



(b)



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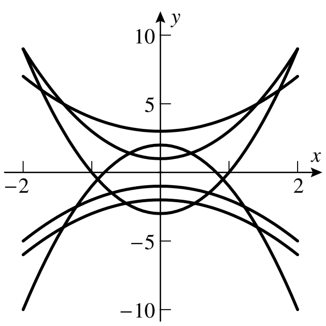
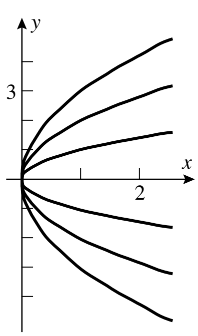


16. **(a)**



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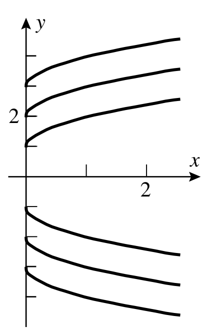

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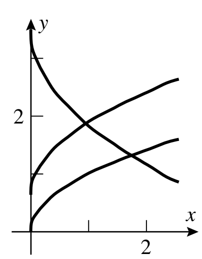
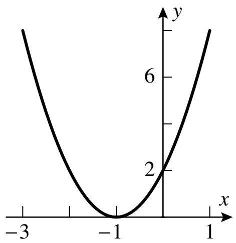
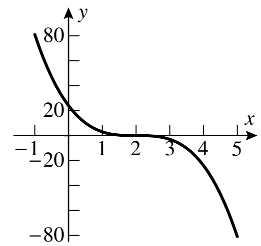


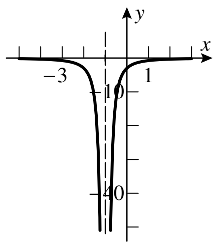
17. **(a)**



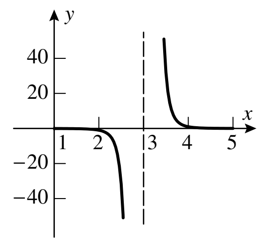
(b)



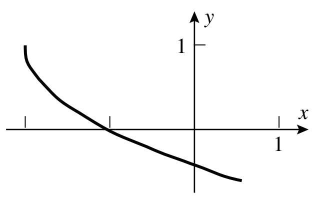
(c)



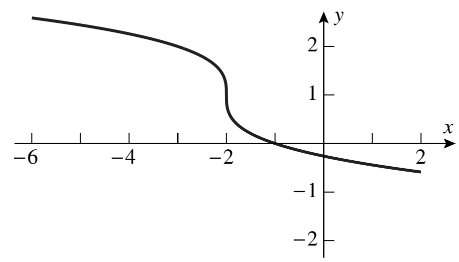
(d)



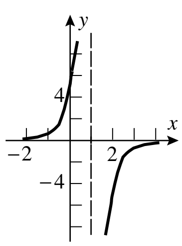
18. **(a)**



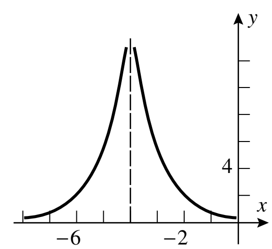
(b)



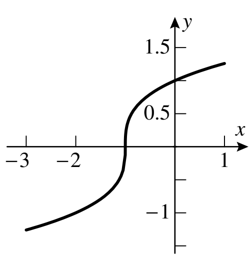
(c)



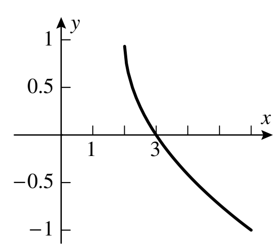
(d)



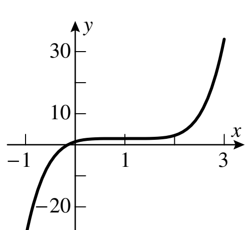
19. **(a)**



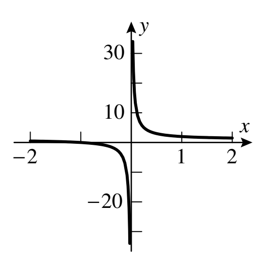
(b)



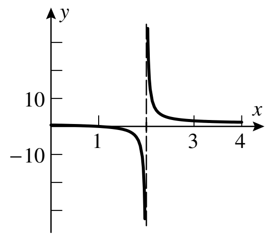
(c)



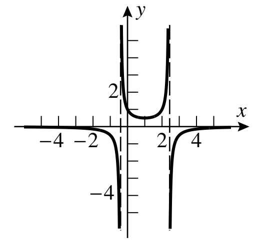
(d)



20. **(a)**



(b)



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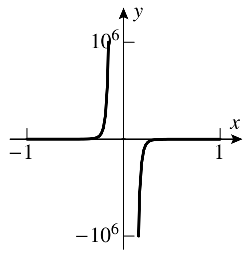

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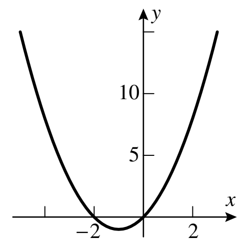
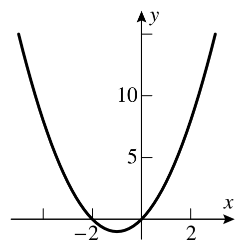
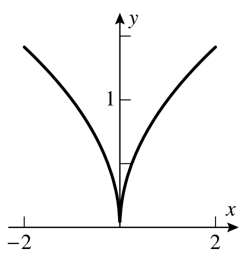


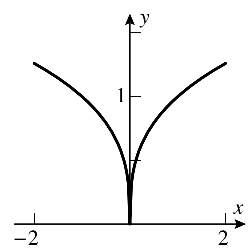
21. $y = x^2 + 2x = (x + 1)^2 - 1$



22. **(a)**



(b)



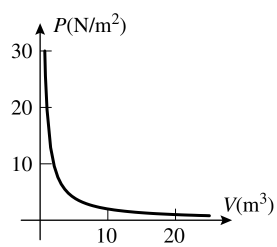
23. **(a)** N·m

(b) k = 20 N·m

(c)

V(L)	0.25	0.5	1.0	1.5	2.0
P (N/m^2)	80×10^3	40×10^3	20×10^3	13.3×10^3	10×10^3

(d)

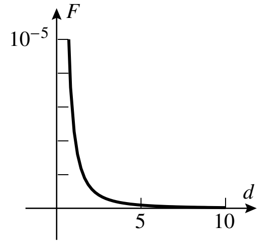


24. If the side of the square base is x and the height of the container is y then $V = x^2y = 100$; minimize $A = 2x^2 + 4xy = 2x^2 + 400/x$. A graphing utility with a zoom feature suggests that the solution is a cube of side $100^{\frac{1}{3}}$ cm.

25. **(a)** $F = k/x^2$ so $0.0005 = k/(0.3)^2$ and $k = 0.000045$ N·m^2.

(b) $F = 0.000005$ N

(c)

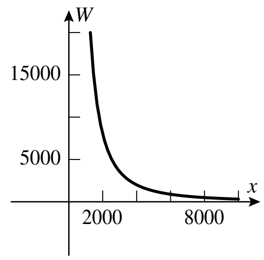


(d) When they approach one another, the force becomes infinite; when they get far apart it tends to zero.

26. **(a)** $2000 = C/(4000)^2$, so $C = 3.2 \times 10^{10}$ lb·mi^2

(b) $W = C/5000^2 = (3.2 \times 10^{10})/(25 \times 10^6) = 1280$ lb.

(c)



(d) No, but W is very small when x is large.

27. **(a)** II; $y = 1$, $x = -1, 2$ **(b)** I; $y = 0$, $x = -2, 3$

(c) IV; $y = 2$ **(d)** III; $y = 0$, $x = -2$

28. The denominator has roots $x = \pm 1$, so $x^2 - 1$ is the denominator. To determine k use the point $(0, -1)$ to get $k = 1$, $y = 1/(x^2 - 1)$.

29. Order the six trigonometric functions as sin, cos, tan, cot, sec, csc:

(a) pos, pos, pos, pos, pos, pos **(b)** neg, zero, undef, zero, undef, neg

(c) pos, neg, neg, neg, neg, pos **(d)** neg, pos, neg, neg, pos, neg

(e) neg, neg, pos, pos, neg, neg **(f)** neg, pos, neg, neg, pos, neg

30. **(a)** neg, zero, undef, zero, undef, neg **(b)** pos, neg, neg, neg, neg, pos

(c) zero, neg, zero, undef, neg, undef **(d)** pos, zero, undef, zero, undef, pos

(e) neg, neg, pos, pos, neg, neg **(f)** neg, neg, pos, pos, neg, neg

31. **(a)** $\sin(\pi - x) = \sin x$; 0.588 **(b)** $\cos(-x) = \cos x$; 0.924

(c) $\sin(2\pi + x) = \sin x$; 0.588 **(d)** $\cos(\pi - x) = -\cos x$; -0.924

(e) $\cos^2 x = 1 - \sin^2 x; 0.655$ **(f)** $\sin^2 2x = 4\sin^2 x \cos^2 x = 4\sin^2 x(1 - \sin^2 x); 0.905$

32. **(a)** $\sin(3\pi + x) = -\sin x$; -0.588

(b) $\cos(-x - 2\pi) = \cos x$; 0.924

(c) $\sin(8\pi + x) = \sin x$; 0.588

(d) $\sin(x/2) = \pm\sqrt{(1 - \cos x)/2}$; use the negative sign for x small and negative; -0.195

(e) $\cos(3\pi + 3x) = -4\cos^3 x + 3\cos x$; -0.384

(f) $\tan^2 x = \dfrac{1 - \cos^2 x}{\cos^2 x}$; 0.172

33. **(a)** $-a$ **(b)** b **(c)** $-c$ **(d)** $\pm\sqrt{1-a^2}$
(e) $-b$ **(f)** $-a$ **(g)** $\pm 2b\sqrt{1-b^2}$ **(h)** $2b^2-1$
(i) $1/b$ **(j)** $-1/a$ **(k)** $1/c$ **(l)** $(1-b)/2$

34. **(a)** The distance is $36/360 = 1/10$th of a great circle, so it is $(1/10)2\pi r = 2,513.27$ mi.
(b) $36/360 = 1/10$

35. If the arc length is x, then solve the ratio $\dfrac{x}{1} = \dfrac{2\pi r}{27.3}$ to get $x \approx 87,458$ km.

36. The distance travelled is equal to the length of that portion of the circumference of the wheel which touches the road, and that is the fraction $225/360$ of a circumference, so a distance of $(225/360)(2\pi)3 = 11.78$ ft

37. The second quarter revolves twice (720°) about its own center.

38. Add r to itself until you exceed $2\pi r$; since $6r < 2\pi r < 7r$, you can cut off 6 pieces of pie, but there's not enough for a full seventh piece. Remaining pie is $\dfrac{2\pi r - 6r}{2\pi r} = 1 - \dfrac{3}{\pi}$ times the original pie.

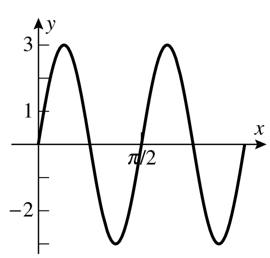
39. **(a)** $y = 3\sin(x/2)$ **(b)** $y = 4\cos 2x$ **(c)** $y = -5\sin 4x$

40. **(a)** $y = 1 + \cos \pi x$ **(b)** $y = 1 + 2\sin x$ **(c)** $y = -5\cos 4x$

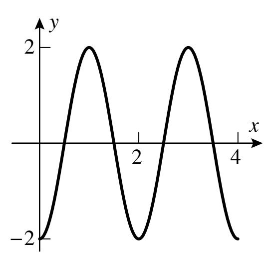
41. **(a)** $y = \sin(x + \pi/2)$ **(b)** $y = 3 + 3\sin(2x/9)$ **(c)** $y = 1 + 2\sin(2(x - \pi/4))$

42. $V = 120\sqrt{2}\sin(120\pi t)$

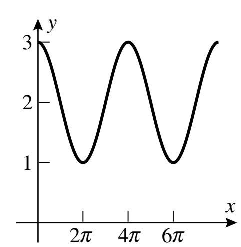
43. **(a)** $3, \pi/2, 0$



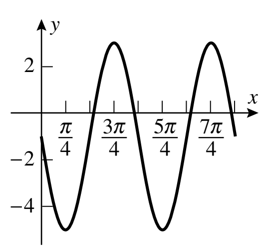
(b) $2, 2, 0$



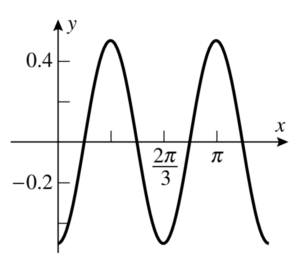
(c) $1, 4\pi, 0$



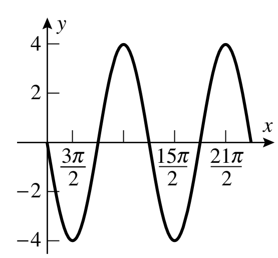
44. **(a)** $4, \pi, 0$



(b) $1/2, 2\pi/3, \pi/3$



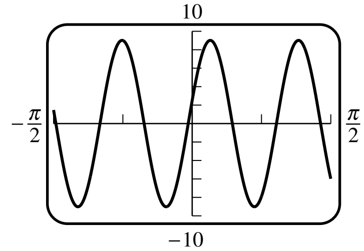
(c) $4, 6\pi, -6\pi$



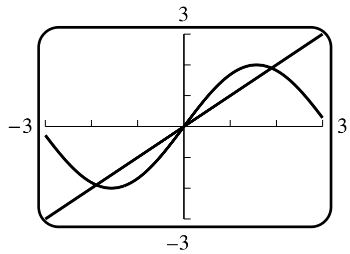
45. **(a)** $A\sin(\omega t + \theta) = A\sin(\omega t)\cos\theta + A\cos(\omega t)\sin\theta = A_1\sin(\omega t) + A_2\cos(\omega t)$
(b) $A_1 = A\cos\theta, A_2 = A\sin\theta$, so $A = \sqrt{A_1^2 + A_2^2}$ and $\theta = \tan^{-1}(A_2/A_1)$.

(c) $A = 5\sqrt{13}/2$, $\theta = \tan^{-1}\dfrac{1}{2\sqrt{3}}$;

$$x = \frac{5\sqrt{13}}{2}\sin\left(2\pi t + \tan^{-1}\frac{1}{2\sqrt{3}}\right)$$



46. three; $x = 0$, $x = \pm 1.8955$

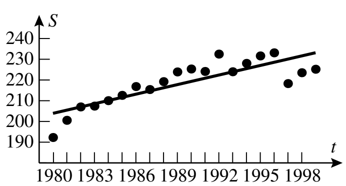


EXERCISE SET 1.7

1. The sum of the squares for the residuals for line I is approximately $1^2 + 1^2 + 1^2 + 0^2 + 2^2 + 1^2 + 1^2 + 1^2 = 10$, and the same for line II is approximately $0^2 + (0.4)^2 + (1.2)^2 + 0^2 + (2.2)^2 + (0.6)^2 + (0.2)^2 + 0^2 = 6.84$; line II is the regression line.

2. **(a)** The data appear to be periodic, so a trigonometric model may be appropriate.
(b) The data appear to lie near a parabola, so a quadratic model may be appropriate.
(c) The data appear to lie near a line, so a linear model may be appropriate.
(d) The data appear to be randomly distributed, so no model seems to be appropriate.

3. Least squares line $S = 1.5388t - 2842.9$, correlation coefficient 0.83409



4. **(a)** $p = 0.0154T + 4.19$ **(b)** 3.42 atm

5. **(a)** Least squares line $p = 0.0146T + 3.98$, correlation coefficient 0.9999
(b) $p = 3.25$ atm **(c)** $T = -272°\text{C}$

6. **(a)** $p = 0.0203\,T + 5.54, r = 0.9999$ **(b)** $T = -273$
(c) $1.05\,p = 0.0203(T)(1.1) + 5.54$ and $p = 0.0203T + 5.54$, subtract to get $.05p = 0.1(0.0203T)$, $p = 2(0.0203T)$. But $p = 0.0203T + 5.54$, equate the right hand sides, get $T = 5.54/0.0203 \approx 273°\text{C}$

7. **(a)** $R = 0.00723T + 1.55$ **(b)** $T = -214°\text{C}$

8. **(a)** $y = 0.0236x + 4.79$ **(b)** $T = -203°\text{C}$

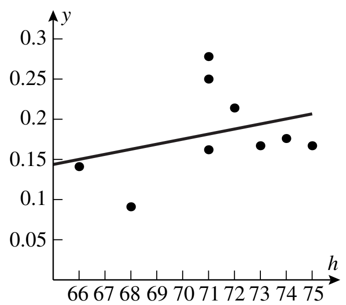
9. **(a)** $S = 0.50179w - 0.00643$ **(b)** $S = 8$, $w = 16$ lb

10. **(a)** $S = 0.756w - 0.0133$

(b) $2S = 0.756(w + 5) - 0.0133$, subtract to get $S = 5(0.756) = 3.78$

11. **(a)** Let h denote the height in inches and y the number of rebounds per minute. Then $y = 0.00630h - 0.266$, $r = 0.313$

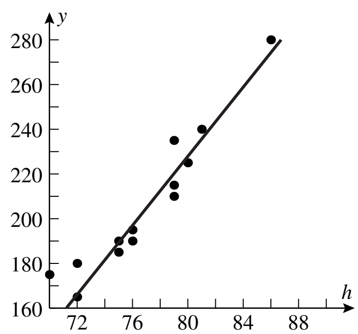
(b)



(c) No, the data points are too widely scattered.

12. **(a)** Let h denote the height in inches and y the weight in pounds. Then $y = 7.73h - 391$

(b)



(c) 259 lb

13. **(a)** $H \approx 20000/110 \approx 181$ km/s/Mly

(b) One light year is 9.408×10^{12} km and

$$t = \frac{d}{v} = \frac{1}{H} = \frac{1}{20\text{km/s/Mly}} = \frac{9.408 \times 10^{18}\text{km}}{20\text{km/s}} = 4.704 \times 10^{17}\text{ s} = 1.492 \times 10^{10}\text{ years.}$$

(c) The Universe would be even older.

14. **(a)** $f = 0.906m + 5.93$ **(b)** $f = 85.7$

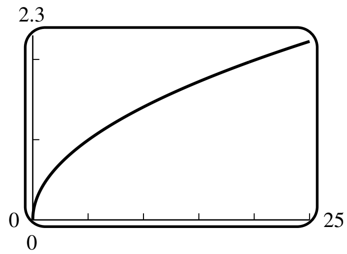
15. **(a)** $P = 0.322t^2 + 0.0671t + 0.00837$ **(b)** $P = 1.43$ cm

16. The population is modeled by $P = 0.0045833t^2 - 16.378t + 14635$; in the year 2000 the population would be $P = 212,200,000$. This is far short; in 1990 the population of the US was approximately 250,000,000.

17. As in Example 4, a possible model is of the form $T = D + A\sin\left[B\left(t - \frac{C}{B}\right)\right]$. Since the longest day has 993 minutes and the shortest has 706, take $2A = 993 - 706 = 287$ or $A = 143.5$. The midpoint between the longest and shortest days is 849.5 minutes, so there is a vertical shift of $D = 849.5$. The period is about 365.25 days, so $2\pi/B = 365.25$ or $B = \pi/183$. Note that the sine function takes the value -1 when $t - \frac{C}{B} = -91.8125$, and T is a minimum at about $t = 0$. Thus the phase shift $\frac{C}{B} \approx 91.5$. Hence $T = 849.5 + 143.5\sin\left[\frac{\pi}{183}t - \frac{\pi}{2}\right]$ is a model for the temperature.

18. As in Example 4, a possible model for the fraction f of illumination is of the form $f = D + A\sin\left[B\left(t - \frac{C}{B}\right)\right]$. Since the greatest fraction of illumination is 1 and the least is 0, $2A = 1$, $A = 1/2$. The midpoint of the fraction of illumination is 1/2, so there is a vertical shift of $D = 1/2$. The period is approximately 30 days, so $2\pi/B = 30$ or $B = \pi/15$. The phase shift is approximately 49/2, so $C/B = 49/2$, and $f = 1/2 + 1/2\sin\left[\frac{\pi}{15}\left(t - \frac{49}{2}\right)\right]$

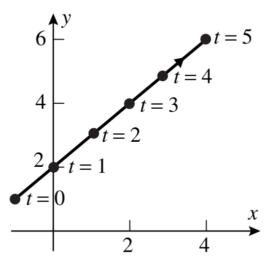
19. $t = 0.445\sqrt{d}$



20. **(a)** $t = 0.373\, r^{1.5}$ **(b)** 238,000 km **(c)** 1.89 days

EXERCISE SET 1.8

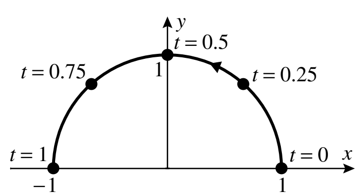
1. **(a)** $x + 1 = t = y - 1$, $y = x + 2$



(c)

t	0	1	2	3	4	5
x	−1	0	1	2	3	4
y	1	2	3	4	5	6

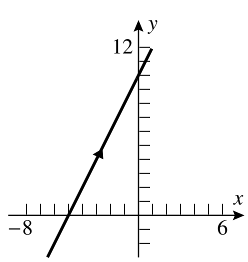
2. **(a)** $x^2 + y^2 = 1$



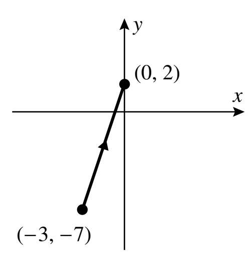
(c)

t	0	0.2500	0.50	0.7500	1
x	1	0.7071	0.00	−0.7071	−1
y	0	0.7071	1.00	0.7071	0

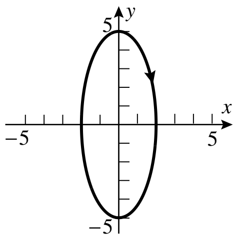
3. $t = (x + 4)/3; y = 2x + 10$



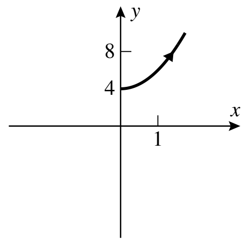
4. $t = x + 3; y = 3x + 2, -3 \le x \le 0$



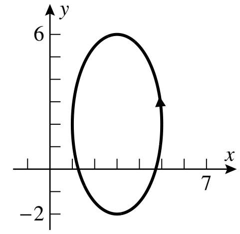
5. $\cos t = x/2$, $\sin t = y/5$; $x^2/4 + y^2/25 = 1$



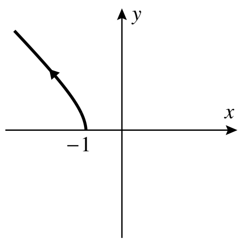
6. $t = x^2$; $y = 2x^2 + 4$, $x \geq 0$



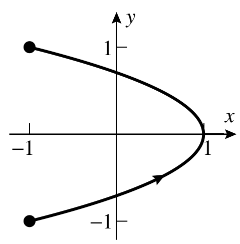
7. $\cos t = (x-3)/2$, $\sin t = (y-2)/4$; $(x-3)^2/4 + (y-2)^2/16 = 1$



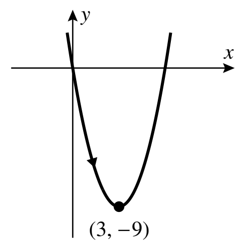
8. $\sec^2 t - \tan^2 t = 1$; $x^2 - y^2 = 1$, $x \leq -1$ and $y \geq 0$



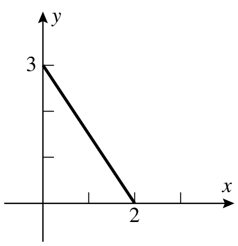
9. $\cos 2t = 1 - 2\sin^2 t$; $x = 1 - 2y^2$, $-1 \leq y \leq 1$



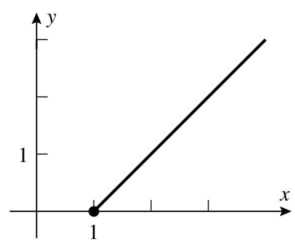
10. $t = (x-3)/4$; $y = (x-3)^2 - 9$



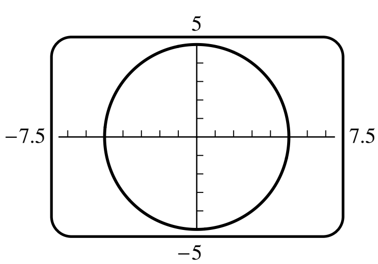
11. $x/2 + y/3 = 1, 0 \leq x \leq 2, 0 \leq y \leq 3$



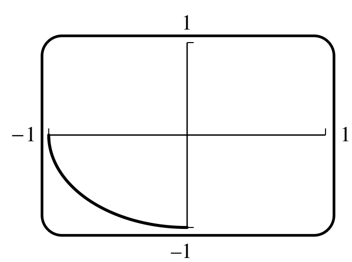
12. $y = x - 1, x \geq 1, y \geq 0$



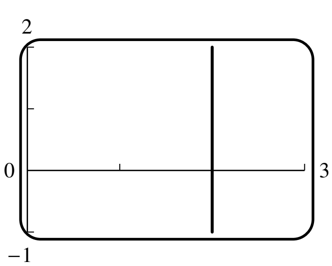
13. $x = 5\cos t$, $y = -5\sin t$, $0 \leq t \leq 2\pi$



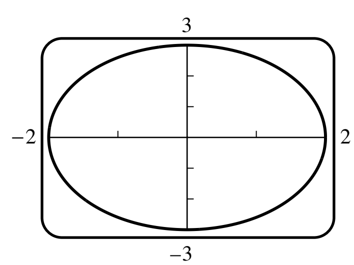
14. $x = \cos t$, $y = \sin t$, $\pi \leq t \leq 3\pi/2$



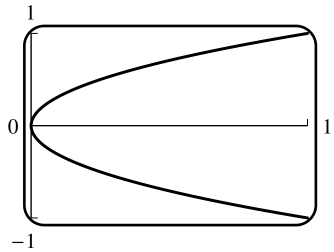
15. $x = 2$, $y = t$



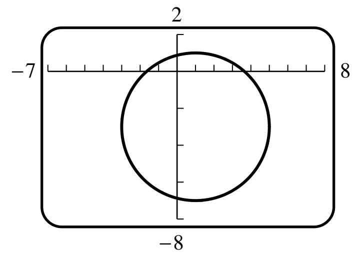
16. $x = 2\cos t$, $y = 3\sin t$, $0 \leq t \leq 2\pi$



17. $x = t^2,\ y = t,\ -1 \le t \le 1$



18. $x = 1 + 4\cos t,\ y = -3 + 4\sin t,\ 0 \le t \le 2\pi$



19. **(a)** IV, because x always increases whereas y oscillates.

(b) II, because $(x/2)^2 + (y/3)^2 = 1$, an ellipse.

(c) V, because $x^2 + y^2 = t^2$ increases in magnitude while x and y keep changing sign.

(d) VI; examine the cases $t < -1$ and $t > -1$ and you see the curve lies in the first, second and fourth quadrants only.

(e) III because $y > 0$.

(f) I; since x and y are bounded, the answer must be I or II; but as t runs, say, from 0 to π, x goes directly from 2 to -2, but y goes from 0 to 1 to 0 to -1 and back to 0, which describes I but not II.

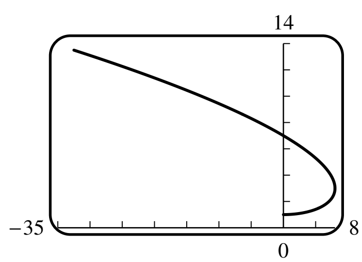
20. **(a)** from left to right **(b)** counterclockwise **(c)** counterclockwise

(d) As t travels from $-\infty$ to -1, the curve goes from (near) the origin in the third quadrant and travels up and left. As t travels from -1 to $+\infty$ the curve comes from way down in the second quadrant, hits the origin at $t = 0$, and then makes the loop clockwise and finally approaches the origin again as $t \to +\infty$.

(e) from left to right

(f) Starting, say, at $(1, 0)$, the curve goes up into the first quadrant, loops back through the origin and into the third quadrant, and then continues the figure-eight.

21. **(a)**



(b)

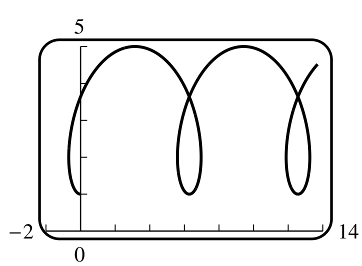
t	0	1	2	3	4	5
x	0	5.5	8	4.5	−8	−32.5
y	1	1.5	3	5.5	9	13.5

(c) $x = 0$ when $t = 0, 2\sqrt{3}$.

(d) for $0 < t < 2\sqrt{2}$

(e) at $t = 2$

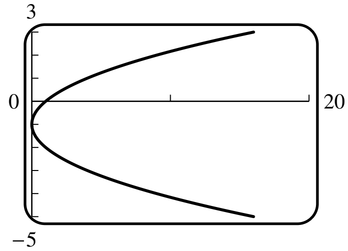
22. **(a)**



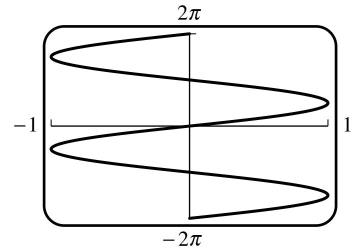
(b) y is always ≥ 1 since $\cos t \le 1$

(c) greater than 5, since $\cos t \ge -1$

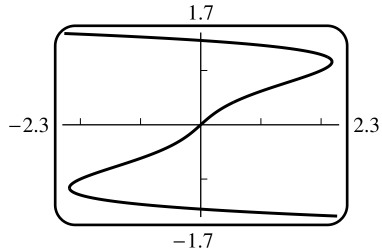
23. **(a)**



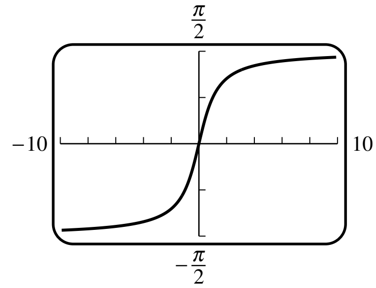
(b)



24. **(a)**



(b)



25. **(a)** Eliminate t to get $\dfrac{x - x_0}{x_1 - x_0} = \dfrac{y - y_0}{y_1 - y_0}$

(b) Set $t = 0$ to get (x_0, y_0); $t = 1$ for (x_1, y_1).

(c) $x = 1 + t$, $y = -2 + 6t$

(d) $x = 2 - t$, $y = 4 - 6t$

26. **(a)** $x = -3 - 2t$, $y = -4 + 5t, 0 \le t \le 1$ **(b)** $x = at$, $y = b(1 - t), 0 \le t \le 1$

27. **(a)** $|R-P|^2 = (x-x_0)^2 + (y-y_0)^2 = t^2[(x_1-x_0)^2 + (y_1-y_0)^2]$ and $|Q-P|^2 = (x_1-x_0)^2 + (y_1-y_0)^2$, so $r = |R - P| = |Q - P|t = qt$.

(b) $t = 1/2$ **(c)** $t = 3/4$

28. $x = 2 + t$, $y = -1 + 2t$

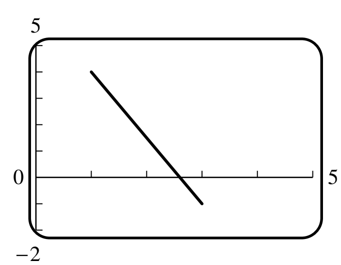
(a) $(5/2, 0)$ **(b)** $(9/4, -1/2)$ **(c)** $(11/4, 1/2)$

29. The two branches corresponding to $-1 \le t \le 0$ and $0 \le t \le 1$ coincide.

30. **(a)** Eliminate $\dfrac{t - t_0}{t_1 - t_0}$ to obtain $\dfrac{y - y_0}{x - x_0} = \dfrac{y_1 - y_0}{x_1 - x_0}$.

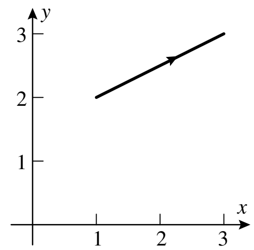
(b) from (x_0, y_0) to (x_1, y_1)

(c) $x = 3 - 2(t - 1)$, $y = -1 + 5(t - 1)$



31. **(a)** $\dfrac{x-b}{a} = \dfrac{y-d}{c}$

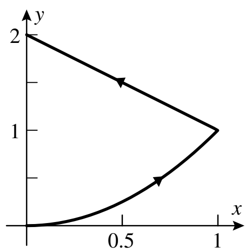
(b)



32. **(a)** If $a = 0$ the line segment is vertical; if $c = 0$ it is horizontal.

(b) The curve degenerates to the point (b, d).

33.



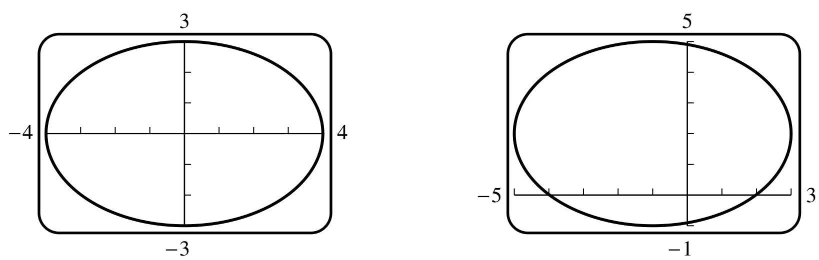
34.

$x = 1/2 - 4t,$	$y = 1/2$	for $0 \le t \le 1/4$
$x = -1/2,$	$y = 1/2 - 4(t - 1/4)$	for $1/4 \le t \le 1/2$
$x = -1/2 + 4(t - 1/2),$	$y = -1/2$	for $1/2 \le t \le 3/4$
$x = 1/2,$	$y = -1/2 + 4(t - 3/4)$	for $3/4 \le t \le 1$

35. **(a)** $x = 4\cos t$, $y = 3\sin t$

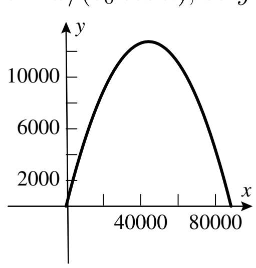
(b) $x = -1 + 4\cos t$, $y = 2 + 3\sin t$

(c)



36. **(a)** $t = x/(v_0 \cos\alpha)$, so $y = x\tan\alpha - gx^2/(2v_0^2\cos^2\alpha)$.

(b)

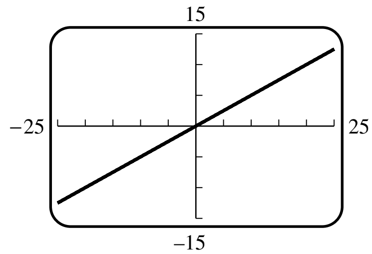


37. **(a)** From Exercise 36, $x = 400\sqrt{2}t$, $y = 400\sqrt{2}t - 4.9t^2$.

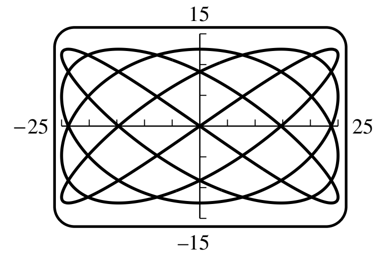
(b) 16,326.53 m

(c) 65,306.12 m

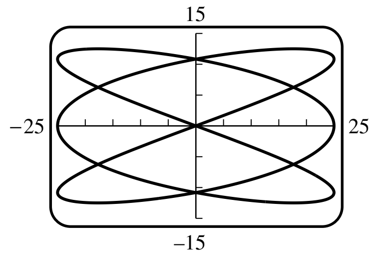
38. **(a)**



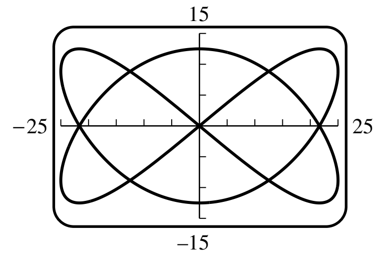
(b)



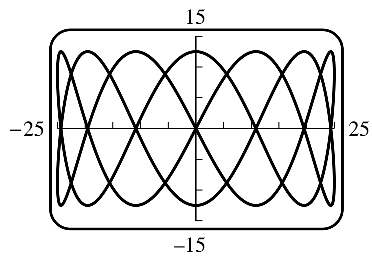
(c)



$a = 3, b = 2$



$a = 2, b = 3$



$a = 2, b = 7$

39. Assume that $a \neq 0$ and $b \neq 0$; eliminate the parameter to get $(x-h)^2/a^2 + (y-k)^2/b^2 = 1$. If $|a| = |b|$ the curve is a circle with center (h,k) and radius $|a|$; if $|a| \neq |b|$ the curve is an ellipse with center (h,k) and major axis parallel to the x-axis when $|a| > |b|$, or major axis parallel to the y-axis when $|a| < |b|$.

(a) ellipses with a fixed center and varying axes of symmetry

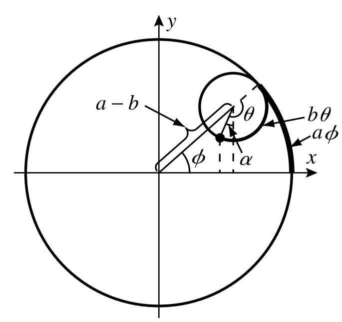
(b) (assume $a \neq 0$ and $b \neq 0$) ellipses with varying center and fixed axes of symmetry

(c) circles of radius 1 with centers on the line $y = x - 1$

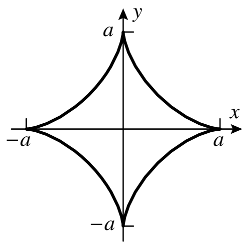
40. Refer to the diagram to get $b\theta = a\phi$, $\theta = a\phi/b$ but

$$\theta - \alpha = \phi + \pi/2 \text{ so } \alpha = \theta - \phi - \pi/2 = (a/b - 1)\phi - \pi/2$$

$$x = (a-b)\cos\phi - b\sin\alpha$$
$$= (a-b)\cos\phi + b\cos\left(\frac{a-b}{b}\right)\phi,$$
$$y = (a-b)\sin\phi - b\cos\alpha$$
$$= (a-b)\sin\phi - b\sin\left(\frac{a-b}{b}\right)\phi.$$



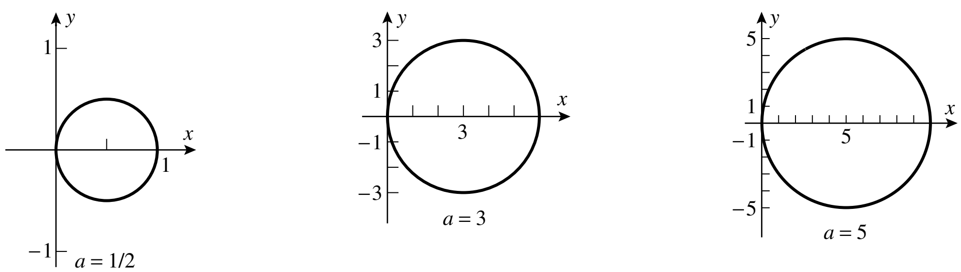
41. **(a)**



(b) Use $b = a/4$ in the equations of Exercise 40 to get
$x = \frac{3}{4}a\cos\phi + \frac{1}{4}a\cos 3\phi$, $y = \frac{3}{4}a\sin\phi - \frac{1}{4}a\sin 3\phi$;
but trigonometric identities yield $\cos 3\phi = 4\cos^3\phi - 3\cos\phi$, $\sin 3\phi = 3\sin\phi - 4\sin^3\phi$,
so $x = a\cos^3\phi$, $y = a\sin^3\phi$.

(c) $x^{2/3} + y^{2/3} = a^{2/3}(\cos^2\phi + \sin^2\phi) = a^{2/3}$

42. **(a)**



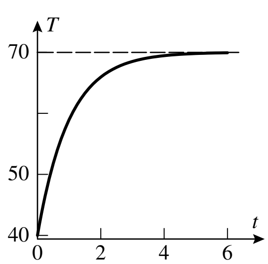
(b)
$$\begin{aligned}(x-a)^2 + y^2 &= (2a\cos^2 t - a)^2 + (2a\cos t\sin t)^2\\ &= 4a^2\cos^4 t - 4a^2\cos^2 t + a^2 + 4a^2\cos^2 t\sin^2 t\\ &= 4a^2\cos^4 t - 4a^2\cos^2 t + a^2 + 4a^2\cos^2 t(1-\cos^2 t) = a^2,\end{aligned}$$
a circle about $(a, 0)$ of radius a

CHAPTER 1 SUPPLEMENTARY EXERCISES

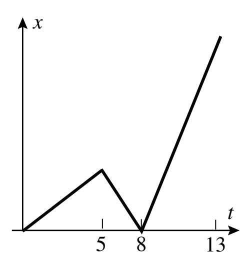
1. 1940-45; the greatest five-year slope

2. **(a)** $f(-1) = 3.3$, $g(3) = 2$
(b) $x = -3, 3$
(c) $x < -2, x > 3$
(d) the domain is $-5 \le x \le 5$ and the range is $-5 \le y \le 4$
(e) the domain is $-4 \le x \le 4.1$, the range is $-3 \le y \le 5$
(f) $f(x) = 0$ at $x = -3, 5$; $g(x) = 0$ at $x = -3, 2$

3.



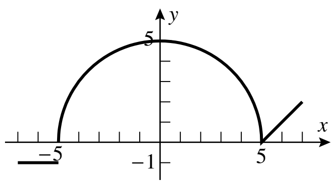
4.



5. If the side has length x and height h, then $V = 8 = x^2h$, so $h = 8/x^2$. Then the cost $C = 5x^2 + 2(4)(xh) = 5x^2 + 64/x$.

6. Assume that the paint is applied in a thin veneer of uniform thickness, so that the quantity of paint to be used is proportional to the area covered. If P is the amount of paint to be used, $P = k\pi r^2$. The constant k depends on physical factors, such as the thickness of the paint, absorption of the wood, etc.

7.



8. Suppose the radius of the uncoated ball is r and that of the coated ball is $r + h$. Then the plastic has volume equal to the difference of the volumes, i.e.
$V = \frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi h[3r^2 + 3rh + h^2]$ in^3.

9. **(a)** The base has sides $(10 - 2x)/2$ and $6 - 2x$, and the height is x, so $V = (6 - 2x)(5 - x)x$ ft^3.
(b) From the picture we see that $x < 5$ and $2x < 6$, so $0 < x < 3$.
(c) 3.57 ft $\times$3.79 ft $\times$1.21 ft

10. $\{x \neq 0\}$ and $\emptyset$ (the empty set)

11. $f(g(x)) = (3x+2)^2+1, g(f(x)) = 3(x^2+1)+2$, so $9x^2+12x+5 = 3x^2+5, 6x^2+12x = 0, x = 0, -2$

12. **(a)** $(3 - x)/x$
(b) no; $f(g(x))$ can be defined at $x = 1$, whereas g, and therefore $f \circ g$, requires $x \neq 1$

13. $1/(2 - x^2)$

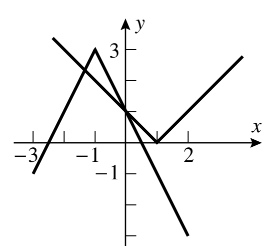
14. $g(x) = x^2 + 2x$

15.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	0	-1	2	1	3	-2	-3	4	-4
$g(x)$	3	2	1	-3	-1	-4	4	-2	0
$(f \circ g)(x)$	4	-3	-2	-1	1	0	-4	2	3
$(g \circ f)(x)$	-1	-3	4	-4	-2	1	2	0	3

16. **(a)** $y = |x - 1|$, $y = |(-x) - 1| = |x + 1|$,
$y = 2|x + 1|$, $y = 2|x + 1| - 3$,
$y = -2|x + 1| + 3$

(b)



17. **(a)** even $\times$ odd = odd
(b) a square is even
(c) even + odd is neither
(d) odd $\times$ odd = even

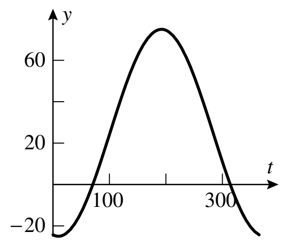
18. **(a)** $y = \cos x - 2\sin x \cos x = (1 - 2\sin x)\cos x$, so $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}$

(b) $\left(\pm\frac{\pi}{2}, 0\right), \left(\pm\frac{3\pi}{2}, 0\right), \left(\frac{\pi}{6}, \sqrt{3}/2\right), \left(\frac{5\pi}{6}, -\sqrt{3}/2\right), \left(-\frac{7\pi}{6}, -\sqrt{3}/2\right), \left(-\frac{11\pi}{6}, \sqrt{3}/2\right)$

19. **(a)** If x denotes the distance from A to the base of the tower, and y the distance from B to the base, then $x^2 + d^2 = y^2$. Moreover $h = x\tan\alpha = y\tan\beta$, so $d^2 = y^2 - x^2 = h^2(\cot^2\beta - \cot^2\alpha)$, $h^2 = \dfrac{d^2}{\cot^2\beta - \cot^2\alpha} = \dfrac{d^2}{1/\tan^2\beta - 1/\tan^2\alpha} = \dfrac{d^2\tan^2\alpha\tan^2\beta}{\tan^2\alpha - \tan^2\beta}$, which yields the result.

(b) 295.72 ft.

20. **(a)**



(b) when $\frac{2\pi}{365}(t - 101) = \frac{3\pi}{2}$, or $t = 374.75$, which is the same date as $t = 9.75$, so during the night of January 10th-11th

(c) from $t = 0$ to $t = 70.58$ and from $t = 313.92$ to $t = 365$ (the same date as $t = 0$) , for a total of about 122 days

21. When $x = 0$ the value of the green curve is higher than that of the blue curve, therefore the blue curve is given by $y = 1 + 2\sin x$.
The points A, B, C, D are the points of intersection of the two curves, i.e. where $1 + 2\sin x = 2\sin(x/2) + 2\cos(x/2)$. Let $\sin(x/2) = p, \cos(x/2) = q$. Then $2\sin x = 4\sin(x/2)\cos(x/2)$, so the equation which yields the points of intersection becomes $1 + 4pq = 2p + 2q$, $4pq - 2p - 2q + 1 = 0, (2p - 1)(2q - 1) = 0$; thus whenever either $\sin(x/2) = 1/2$ or $\cos(x/2) = 1/2$, i.e. when $x/2 = \pi/6, 5\pi/6, \pm\pi/3$. Thus A has coordinates $(-2\pi/3, 1 - \sqrt{3})$, B has coordinates $(\pi/3, 1 + \sqrt{3})$, C has coordinates $(2\pi/3, 1 + \sqrt{3})$, and D has coordinates $(5\pi/3, 1 - \sqrt{3})$.

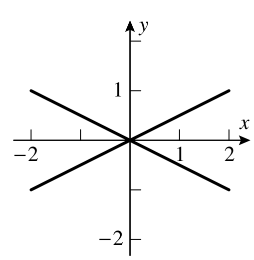
22. Let $y = A + B\sin(at + b)$. Since the maximum and minimum values of y are 35 and 5, $A + B = 35$ and $A - B = 5$, $A = 20$, $B = 15$. The period is 12 hours, so $12a = 2\pi$ and $a = \pi/6$. The maximum occurs at $t = 2$, so $1 = \sin(2a + b) = \sin(\pi/3 + b)$, $\pi/3 + b = \pi/2$, $b = \pi/2 - \pi/3 = \pi/6$ and $y = 20 + 15\sin(\pi t/6 + \pi/6)$.

23. **(a)** The circle of radius 1 centered at (a, a^2); therefore, the family of all circles of radius 1 with centers on the parabola $y = x^2$.

(b) All parabolas which open up, have latus rectum equal to 1 and vertex on the line $y = x/2$.

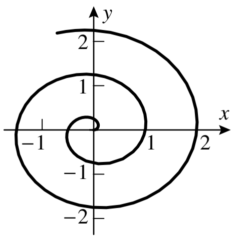
24. **(a)** $x = f(1 - t), y = g(1 - t)$

25.



26. Let $y = ax^2 + bx + c$. Then $4a + 2b + c = 0, 64a + 8b + c = 18, 64a - 8b + c = 18$, from which $b = 0$ and $60a = 18$, or finally $y = \frac{3}{10}x^2 - \frac{6}{5}$.

27.



28. **(a)** $R = R_0$ is the R-intercept, $R_0 k$ is the slope, and $T = -1/k$ is the T-intercept

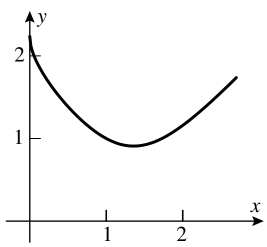
(b) $-1/k = -273$, or $k = 1/273$

(c) $1.1 = R_0(1 + 20/273)$, or $R_0 = 1.025$

(d) $T = 126.55°\text{C}$

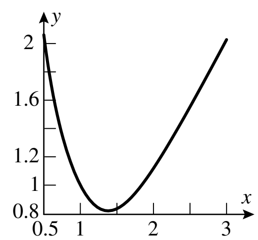
29. $d = \sqrt{(x-1)^2 + (\sqrt{x} - 2)^2}$;

$d = 9.1$ at $x = 1.358094$



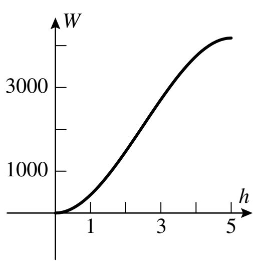
30. $d = \sqrt{(x-1)^2 + 1/x^2}$;

$d = 0.82$ at $x = 1.380278$



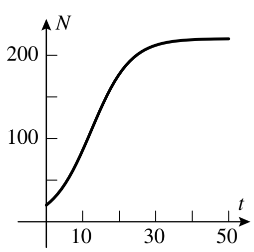
31. $w = 63.9V$, $w = 63.9\pi h^2(5/2 - h/3)$; $h = 0.48$ ft when $w = 108$ lb

32. **(a)**



(b) $w = 63.9\pi h^2(5/2 - h/3)$; at $h = 5/2$, $w = 2091.12$ lb

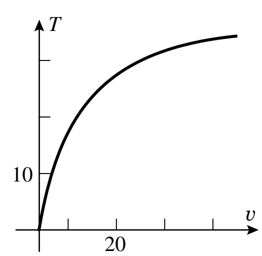
33. **(a)**



(b) $N = 80$ when $t = 9.35$ yrs

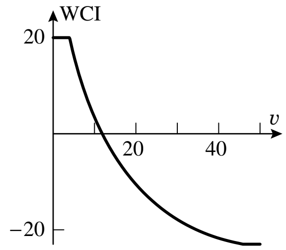
(c) 220 sheep

34. **(a)**



(b) $T = 17°\text{F}, 27°\text{F}, 32°\text{F}$

35. **(a)**



(b) $T = 3°\text{F}, -11°\text{F}, -18°\text{F}, -22°\text{F}$

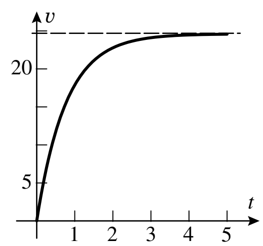
(c) $v = 35, 19, 12, 7$ mi/h

36. The domain is the set of all x, the range is $-0.1746 \le y \le 0.1227$.

37. The domain is the set $-0.7245 \le x \le 1.2207$, the range is $-1.0551 \le y \le 1.4902$.

38. **(a)** The potato is done in the interval $27.65 < t < 32.71$.
(b) 91.54 min.

39. **(a)**



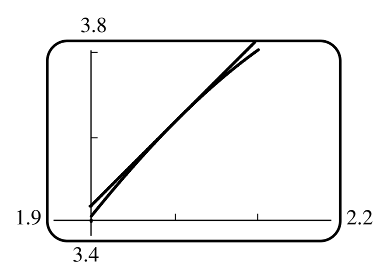
(b) As $t \to \infty$, $(0.273)^t \to 0$, and thus $v \to 24.61$ ft/s.
(c) For large t the velocity approaches c.
(d) No; but it comes very close (arbitrarily close).
(e) 3.013 s

40. **(a)** $y = -0.01716428571x + 1.433827619$

41. **(a)**

1.90	1.92	1.94	1.96	1.98	2.00	2.02	2.04	2.06	2.08	2.10
3.4161	3.4639	3.5100	3.5543	3.5967	3.6372	3.6756	3.7119	3.7459	3.7775	3.8068

(b) $y = 1.9589x - 0.2910$
(c) $y - 3.6372 = 1.9589(x - 2)$, or $y = 1.9589x - 0.2806$
(d) As one zooms in on the point $(2, f(2))$ the two curves seem to converge to one line.



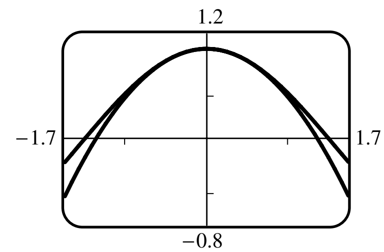
42. **(a)**

−0.10	−0.08	−0.06	−0.04	−0.02	0.00	0.02	0.04	0.06	0.08	0.10
0.9950	0.9968	0.9982	0.9992	0.9998	1.0000	0.9998	0.9992	0.9982	0.9968	0.9950

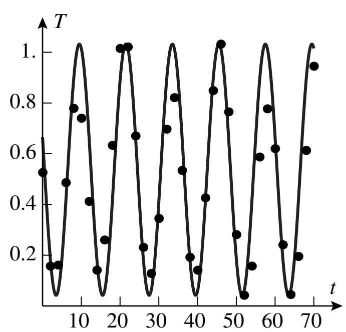
(b) $y = -\frac{1}{2}x^2 + 1$

(c) $y = -\frac{1}{2}x^2 + 1$

(d) As one zooms in on the point $(0, f(0))$ the two curves seem to converge to one curve.



43. The data are periodic, so it is reasonable that a trigonometric function might approximate them. A possible model is of the form $T = D + A\sin\left[B\left(t - \frac{C}{B}\right)\right]$. Since the highest level is 1.032 meters and the lowest is 0.045, take $2A = 1.032 - 0.042 = 0.990$ or $A = 0.495$. The midpoint between the lowest and highest levels is 0.537 meters, so there is a vertical shift of $D = 0.537$. The period is about 12 hours, so $2\pi/B = 12$ or $B = \pi/6$. The phase shift $\frac{C}{B} \approx 6.5$. Hence $T = 0.537 + 0.495\sin\left[\frac{\pi}{6}(t - 6.5)\right]$ is a model for the temperature.



CHAPTER 1 HORIZON MODULE

1. **(a)** $0.25, 6.25\times10^{-2}, 3.91\times10^{-3}, 1.53\times10^{-5}, 2.32\times10^{-10}, 5.42\times10^{-20}, 2.94\times10^{-39}, 8.64\times10^{-78}, 7.46\times10^{-155}, 5.56\times10^{-309}$;
$1, 1, 1, 1, 1, 1, 1, 1, 1, 1$;
$4, 16, 256, 65536, 4.29\times10^{9}, 1.84\times10^{19}, 3.40\times10^{38}, 1.16\times10^{77}, 1.34\times10^{154}, 1.80\times10^{308}$

2. $1, 3, 2.3333333, 2.23809524, 2.23606890, 2.23606798, \ldots$

3. **(a)** $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$ **(b)** $y_n = \frac{1}{2^n}$

4. **(a)** $y_{n+1} = 1.05y_n$

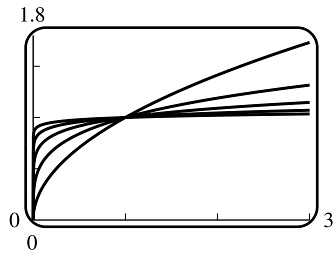
(b) y_0 =\$1000, y_1 =\$1050, y_2 =\$1102.50, y_3 =\$1157.62, y_4 =\$1215.51, y_5 =\$1276.28

(c) $y_{n+1} = 1.05y_n$ for $n \geq 1$

(d) $y_n = (1.05)^n 1000$; y_{15} =\$2078.93

5. **(a)** $x^{1/2}, x^{1/4}, x^{1/8}, x^{1/16}, x^{1/32}$

(b) They tend to the horizontal line $y = 1$, with a hole at $x = 0$.



6. **(a)** $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{5}$ $\frac{5}{8}$ $\frac{8}{13}$ $\frac{13}{21}$ $\frac{21}{34}$ $\frac{34}{55}$ $\frac{55}{89}$ $\frac{89}{144}$

(b) 1, 2, 3, 5, 8, 13, 21, 34, 55, 89;
each new numerator is the sum of the previous two numerators.

(c) $\frac{144}{233}, \frac{233}{377}, \frac{377}{610}, \frac{610}{987}, \frac{987}{1597}, \frac{1597}{2584}, \frac{2584}{4181}, \frac{4181}{6765}, \frac{6765}{10946}, \frac{10946}{17711}$

(d) $F_0 = 1$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.

(e) the positive solution

7. **(a)** $y_1 = cr$, $y_2 = cy_1 = cr^2$, $y_3 = cr^3$, $y_4 = cr^4$ **(b)** $y_n = cr^n$

(c) If $r = 1$ then $y_n = c$ for all n; if $r < 1$ then y_n tends to zero; if $r > 1$, then y_n gets ever larger (tends to $+\infty$).

8. The first point on the curve is $(c, kc(1 - c))$, so $y_1 = kc(1 - c)$ and hence y_1 is the first iterate. The point on the line to the right of this point has equal coordinates (y_1, y_1), and so the point above it on the curve has coordinates $(y_1, ky_1(1 - y_1))$; thus $y_2 = ky_1(1 - y_1)$, and y_2 is the second iterate, etc.

9. **(a)** $0.261, 0.559, 0.715, 0.591, 0.701$

(b) It appears to approach a point somewhere near 0.65.